Extracting the redshift factor in binary black hole simulations

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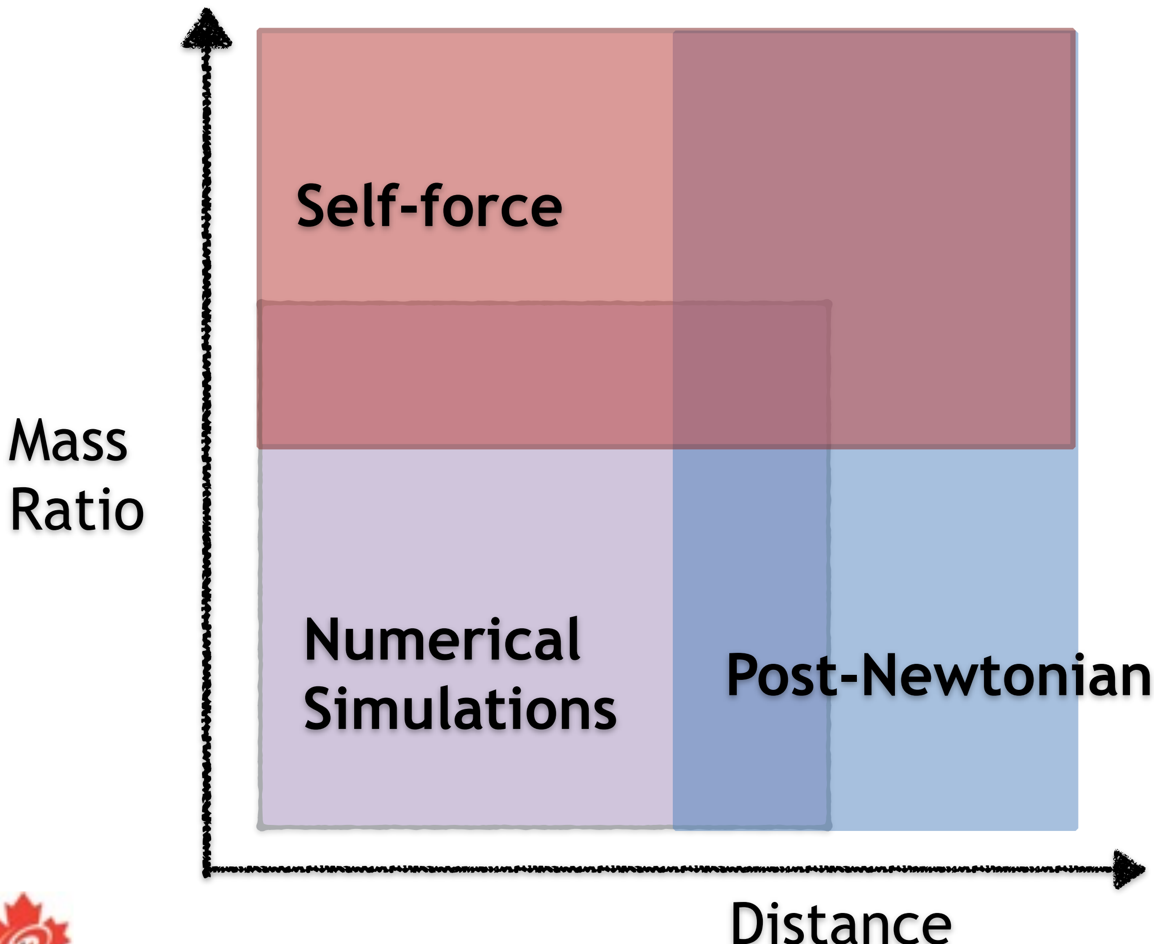
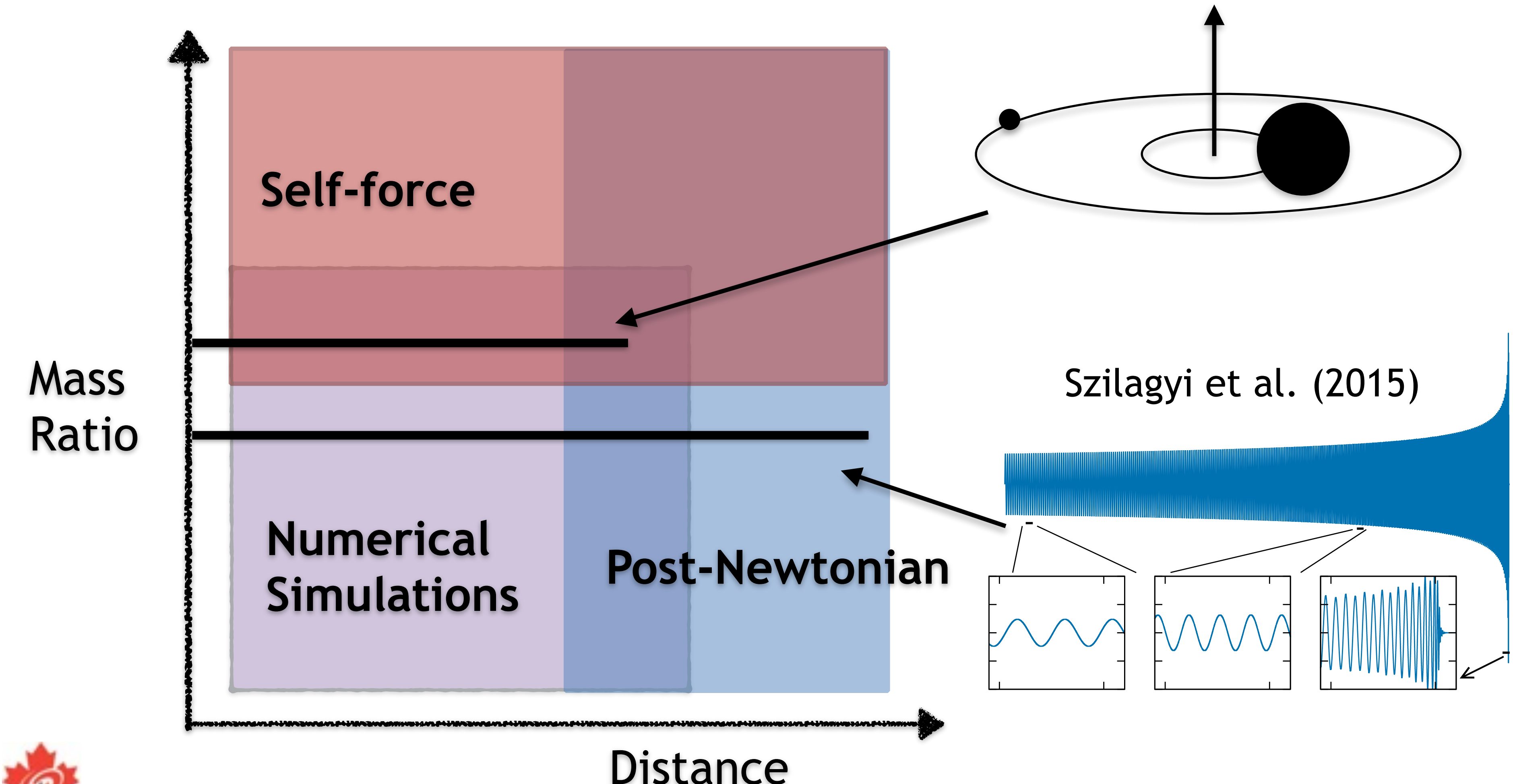
Two body problem



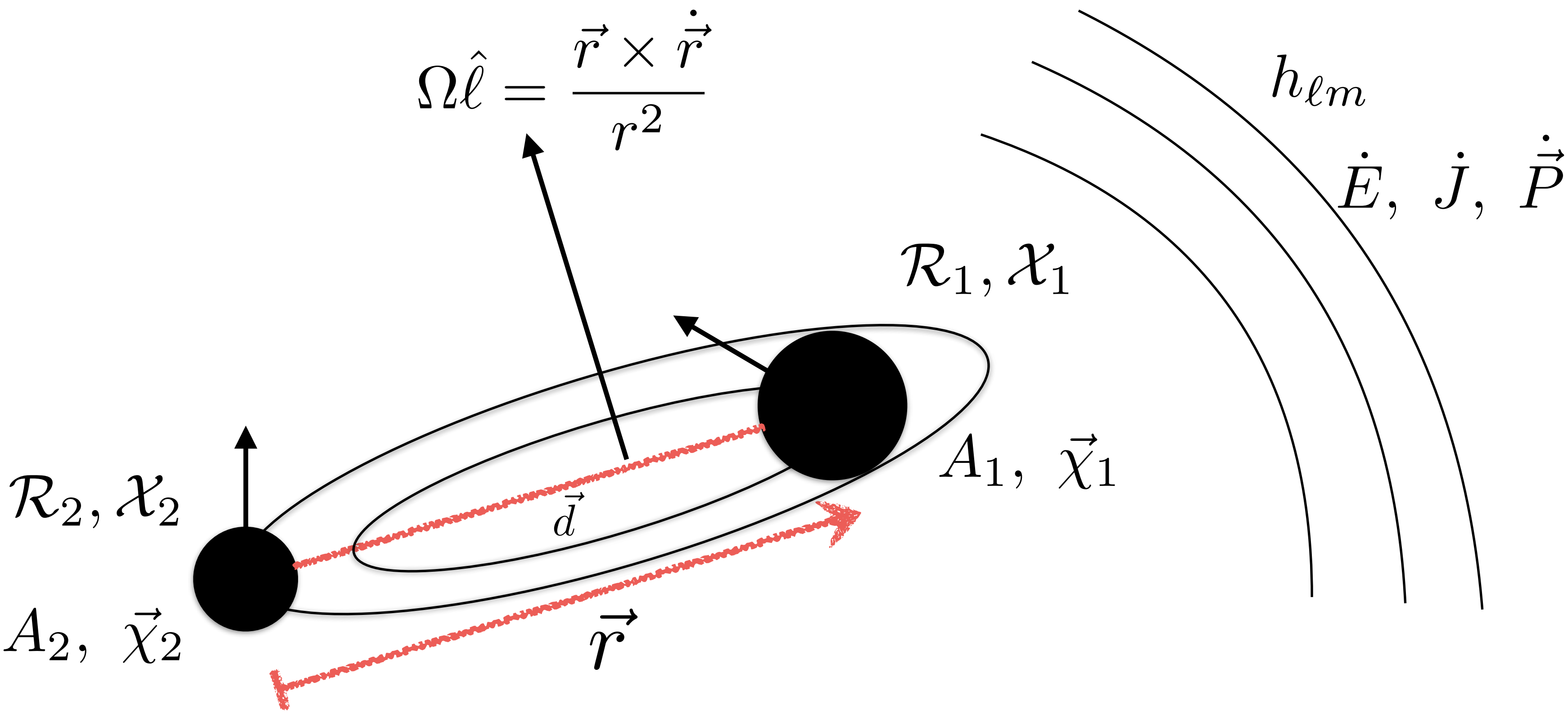
Two body problem



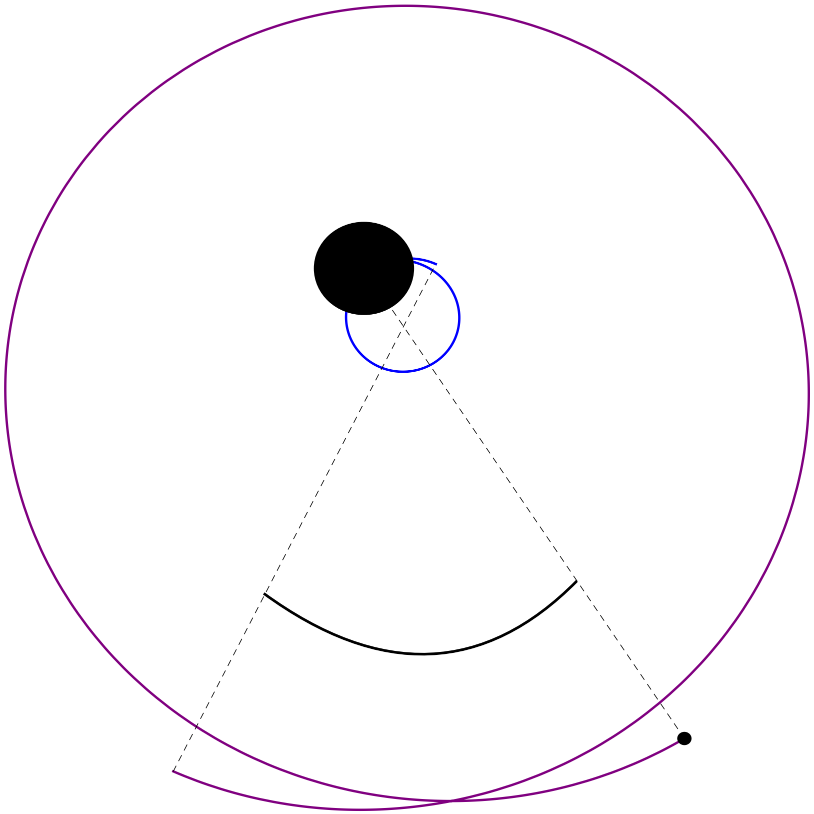
Comparisons between SF and NR

- Want to compare (pseudo-)invariants
- Independent verification of SF predictions
- Discovery: test regime of validity
- Compute higher order SF effects
- Calibrate analytic waveform models

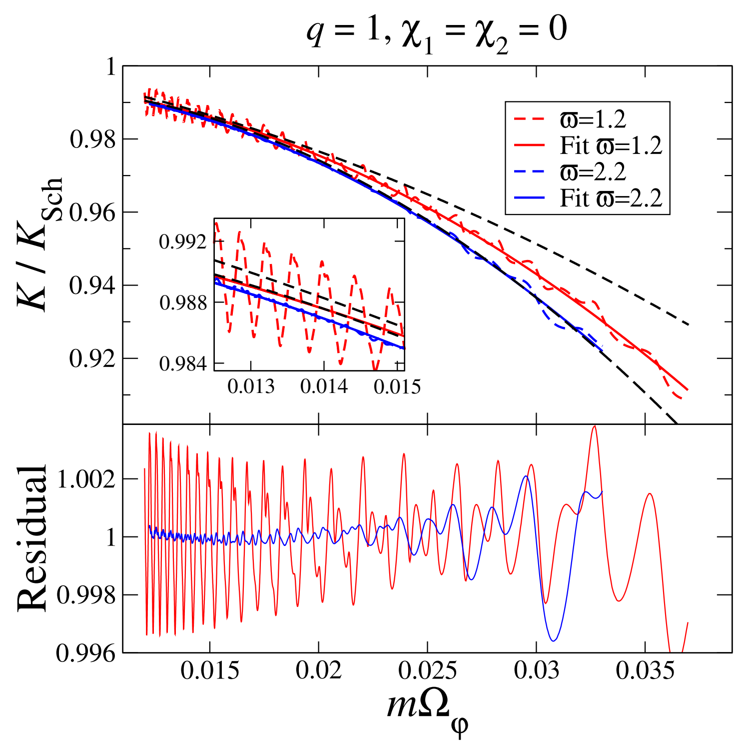
What do we get from NR?



Periastron Precession

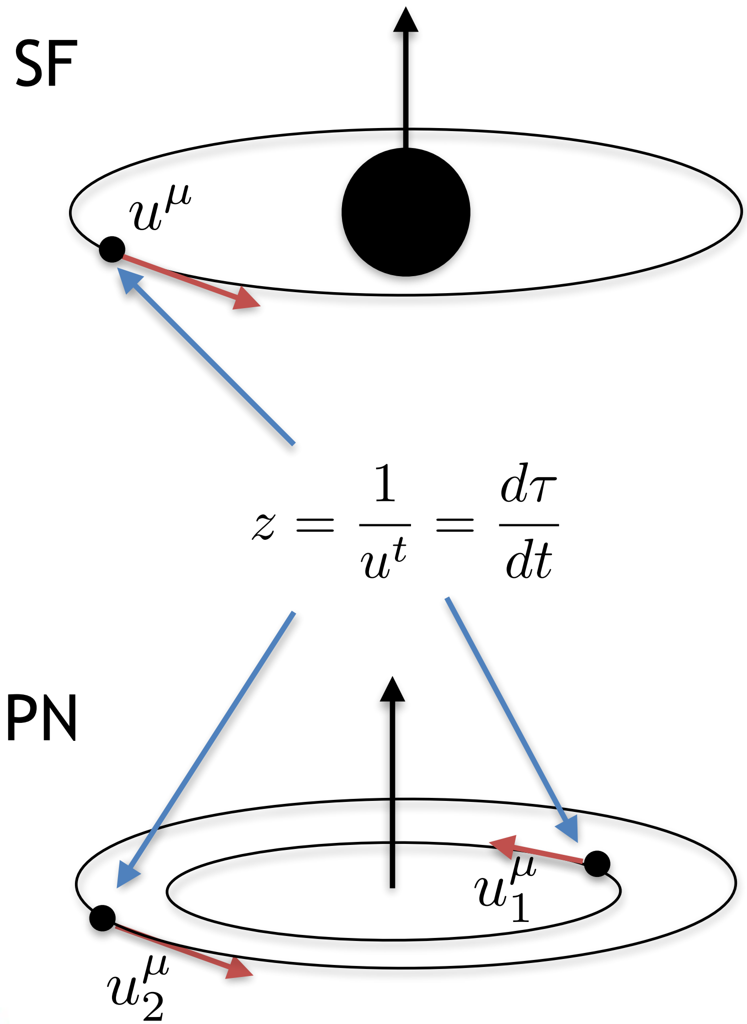


$$\Delta\Phi = 2\pi\left(\frac{\Omega_\phi}{\Omega_r} - 1\right)$$



Le Tiec et al. (2013)

Redshift factor z

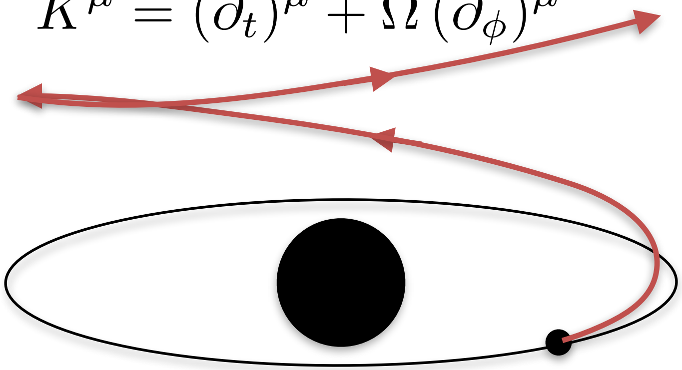


$$z = \frac{1}{u^t} = \frac{d\tau}{dt}$$

- Invariant quantity in SF and PN theories
- Wealth of connections: SF, PN, EOB
- Sims have extended bodies
- Interface w/ NR: connect to surface grav

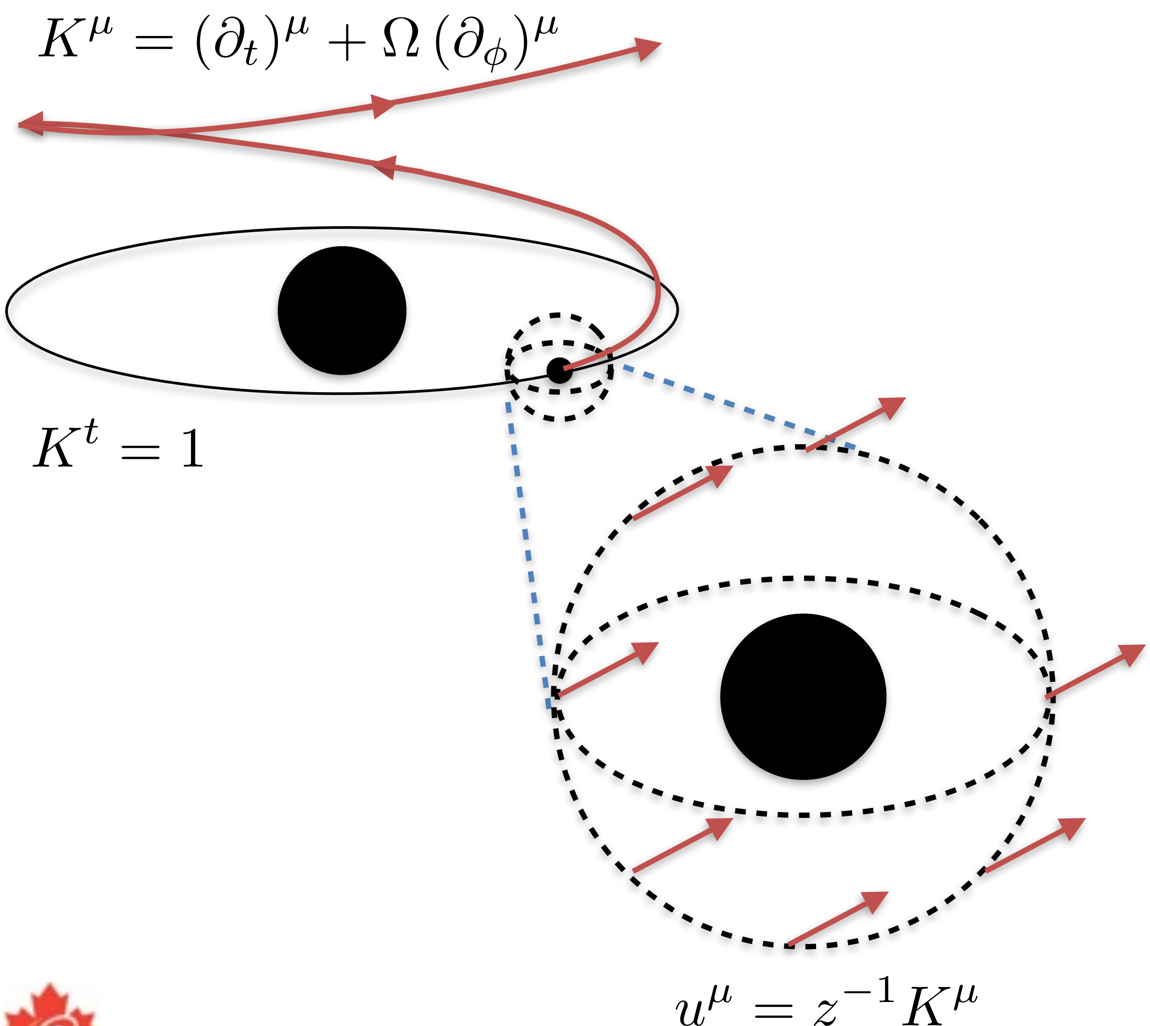
Redshift and surface gravity

$$K^{\mu} = (\partial_t)^{\mu} + \Omega\,(\partial_{\phi})^{\mu}$$

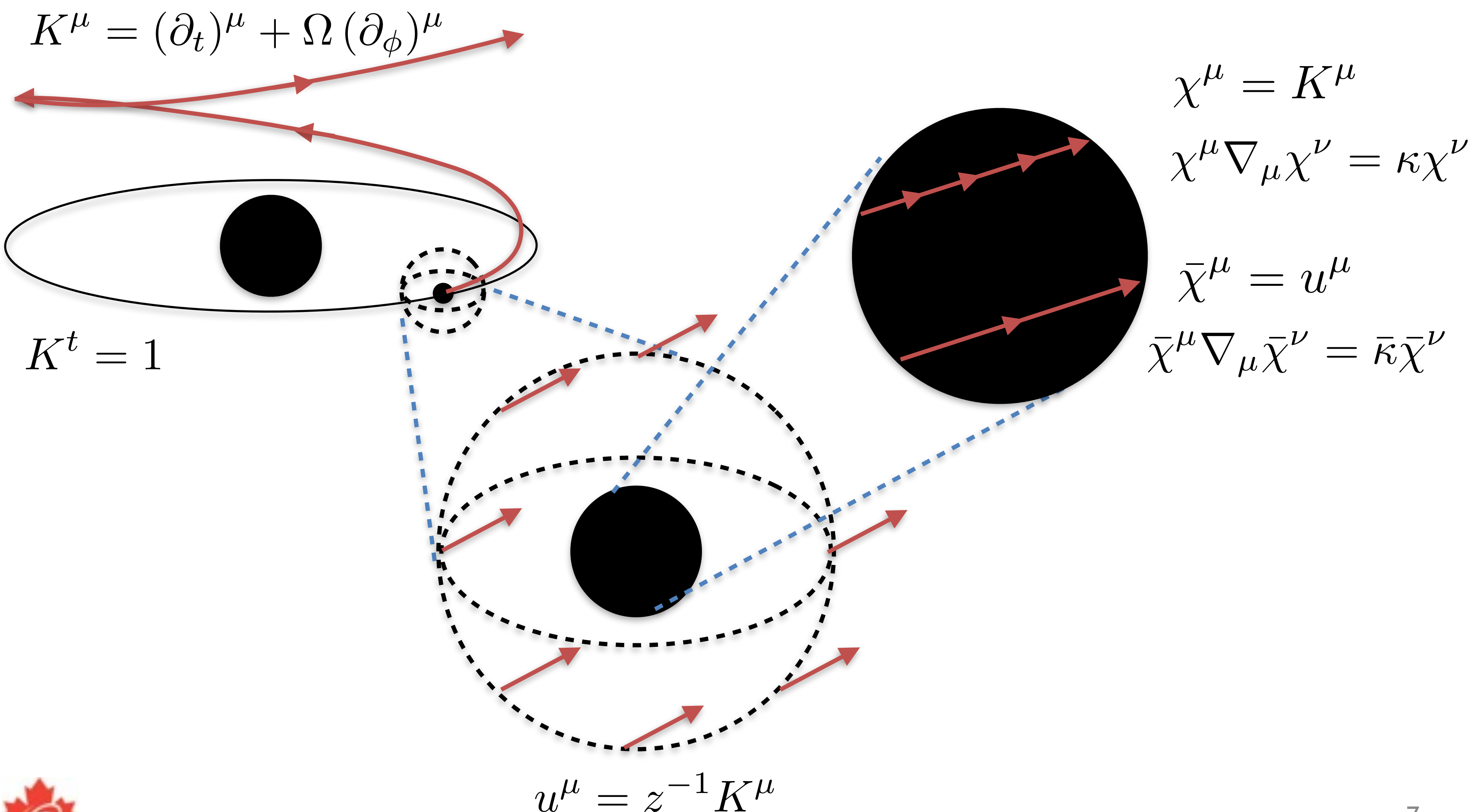


$$K^t = 1$$

Redshift and surface gravity

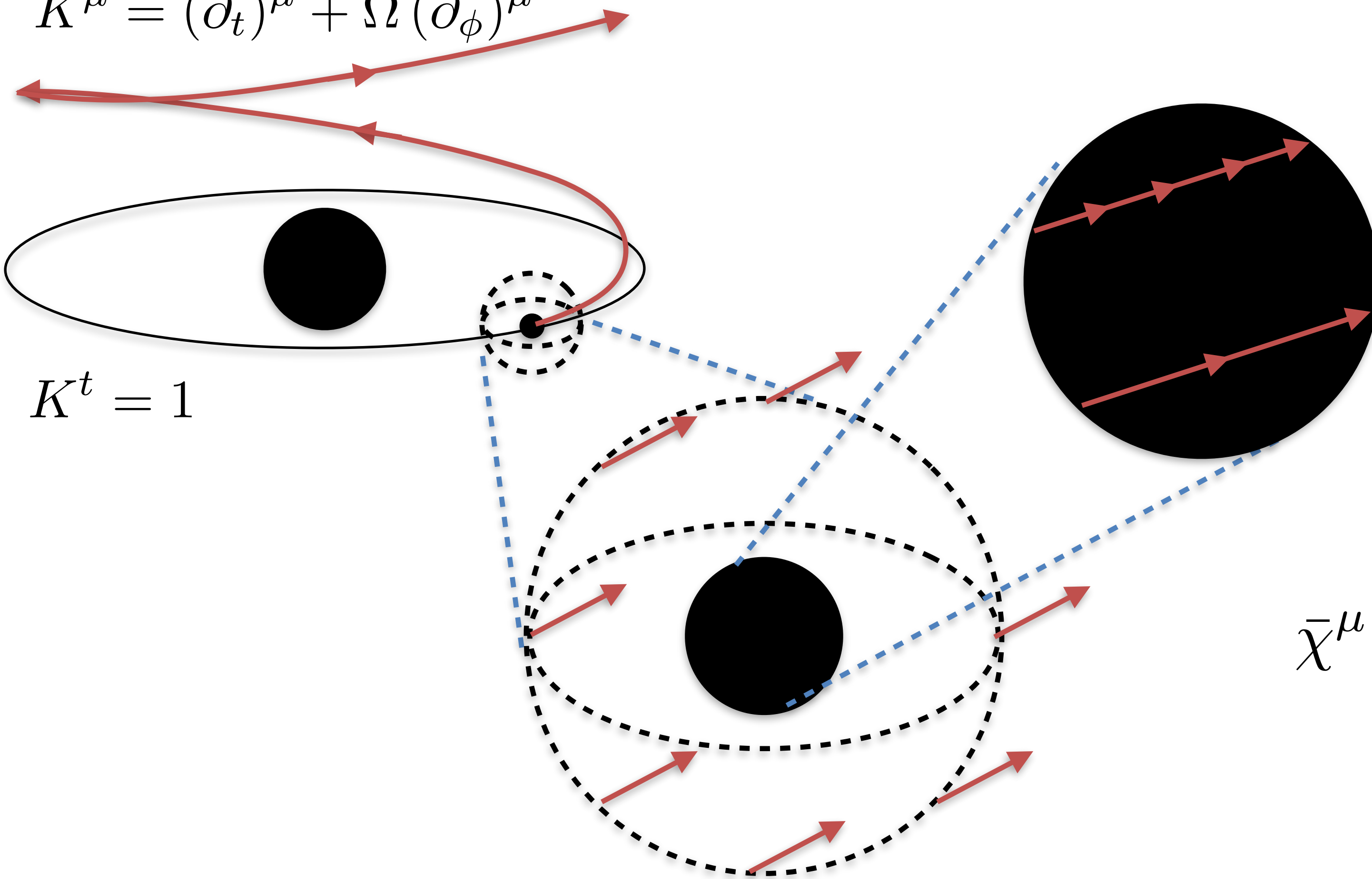


Redshift and surface gravity



Redshift and surface gravity

$$K^\mu = (\partial_t)^\mu + \Omega\,(\partial_\phi)^\mu$$



$$K^t = 1$$

$$u^\mu = z^{-1} K^\mu$$

$$\chi^\mu = K^\mu$$

$$\chi^\mu \nabla_\mu \chi^\nu = \kappa \chi^\nu$$

$$\bar{\chi}^\mu = u^\mu$$

$$\bar{\chi}^\mu \nabla_\mu \bar{\chi}^\nu = \bar{\kappa} \bar{\chi}^\nu$$

$$\bar{\chi}^\mu = z^{-1} \chi^\mu$$

$$z = \frac{\kappa}{\bar{\kappa}}$$

Redshift in NR

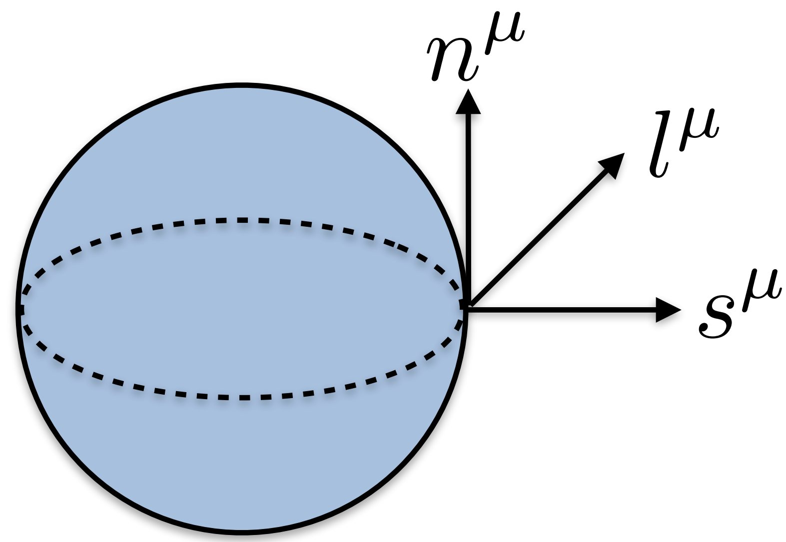
- Normalization absent
- Consider rescaling

$$l^\mu \to \alpha l^\mu \quad \kappa_{(l)} \to \alpha \kappa_{(l)}$$

- Rescaling invariant z

$$z = \frac{\kappa_{(l)}}{l^t \bar{\kappa}} \qquad z \to z$$

- Don't access EH: use AH



$$\langle z \rangle = \frac{\int dA \; z(\theta^B)}{A}$$

Approximate HKV

- Quasicircular inspirals only have approximate HKV

$$\nabla_{(\mu} l_{\nu)} \neq 0$$

- Generators shear, horizon grows

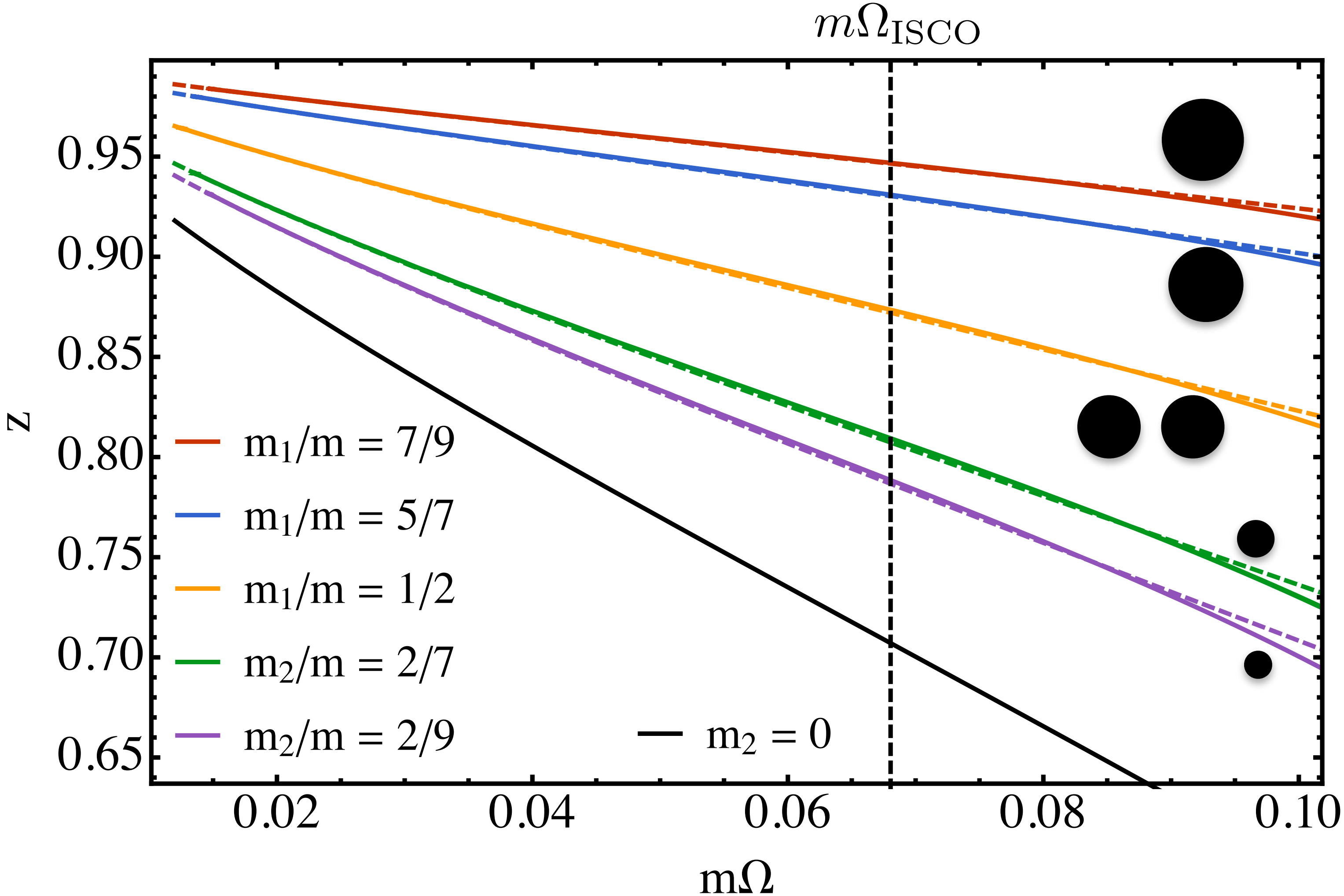
$$\sigma_{\mu\nu}\sigma^{\mu\nu} \sim \dot{m}$$

$$\dot{m} \sim 10^{-9} \implies |\sigma| \sim 10^{-4} - 10^{-5}$$

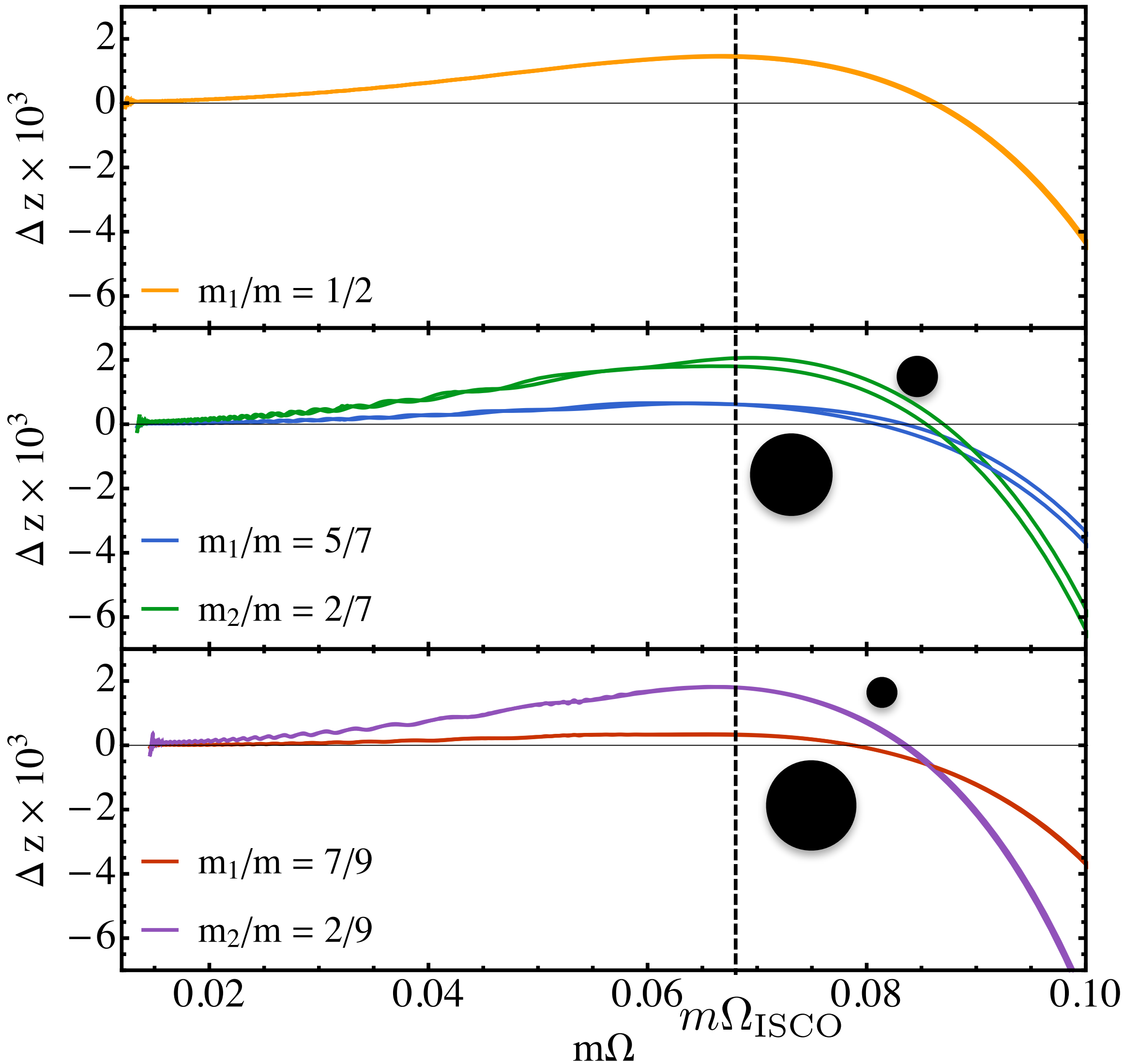
- Other errors: no corotation, nonadiabatic

$$\frac{\dot{\Omega}}{2\Omega^2} \sim 10^{-2} - 10^{-4} \qquad \Omega_H^2 \sim \Omega^2 \sim 10^{-2} - 10^{-4}$$

Redshift factor in NR



Redshift vs PN



First law of binary black holes

- First law: thermodynamic relation for BHs

$$\delta M - \Omega_H \delta J = \kappa \frac{\delta A}{8\pi}$$

- Modified relations for circular binaries

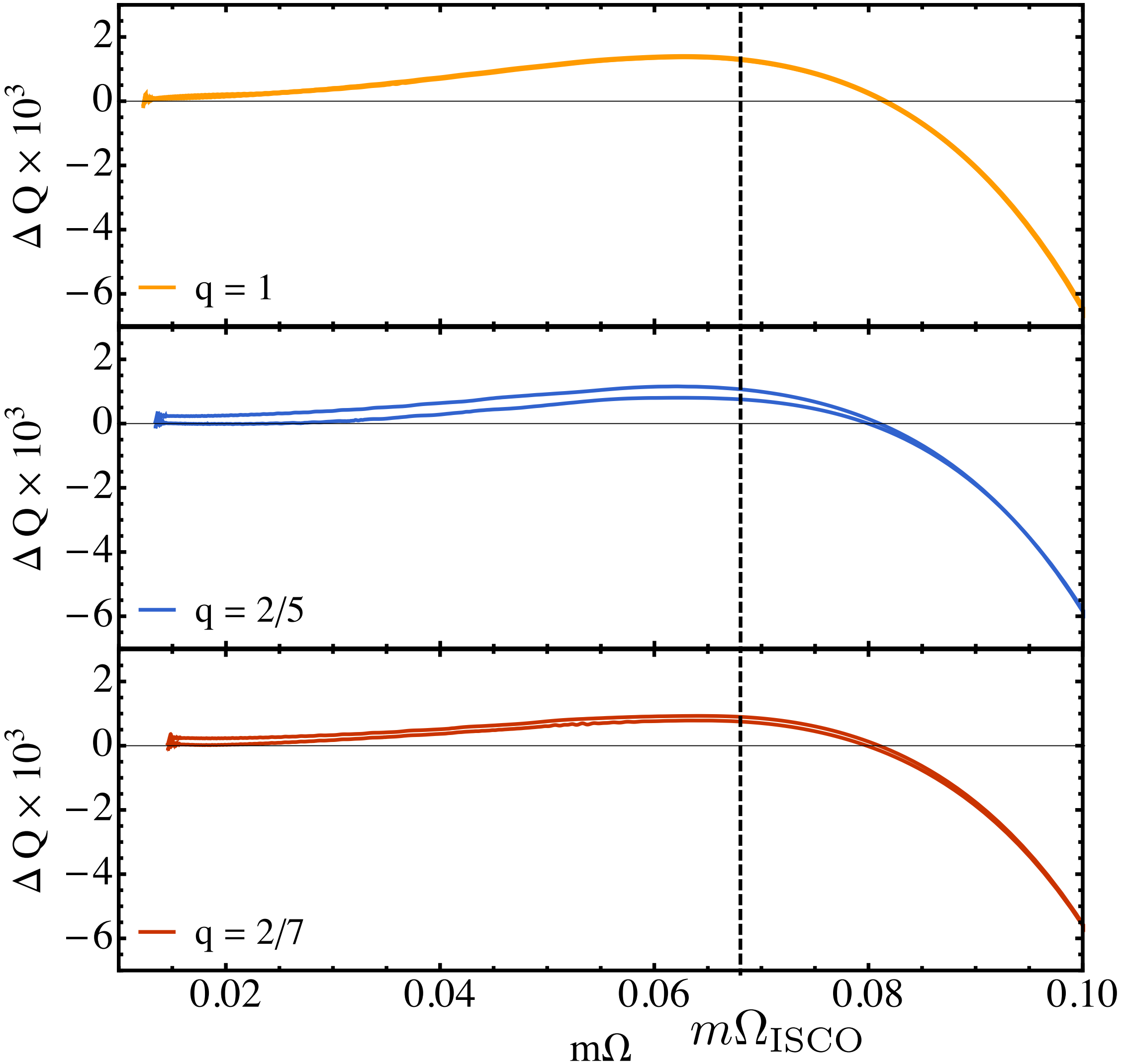
$$\delta M - \Omega_H \delta J = \kappa_1 \frac{\delta A_1}{8\pi} + \kappa_2 \frac{\delta A_2}{8\pi} = z_1 \delta m_1 + z_2 \delta m_2$$

$$Q = M - 2\Omega J = z_1 m_1 + z_2 m_2$$

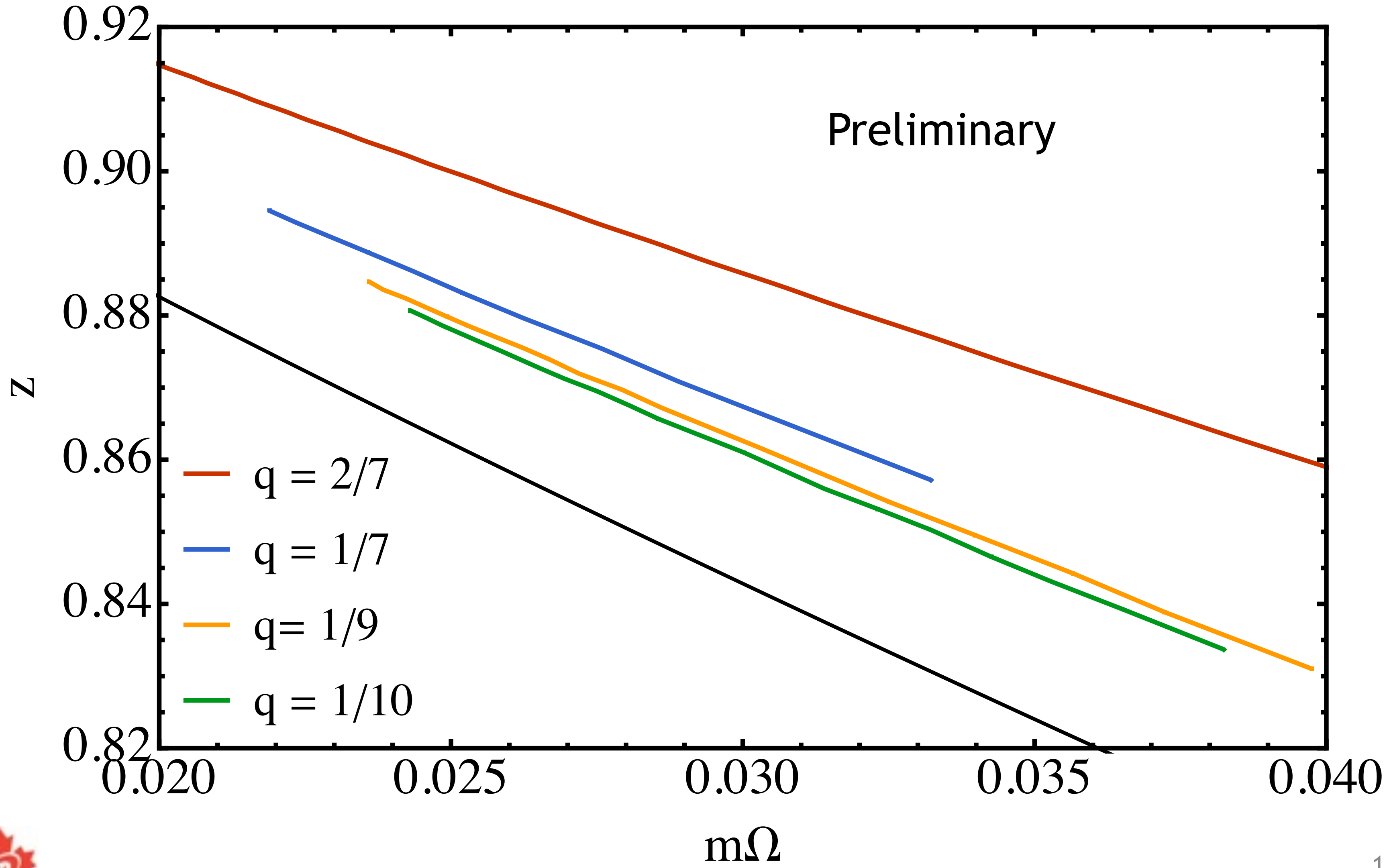
- Connect local and global properties, lower and higher orders in SF
- Used in SF, PN, EOB
- Can test with our numerical z

First law of binary black holes

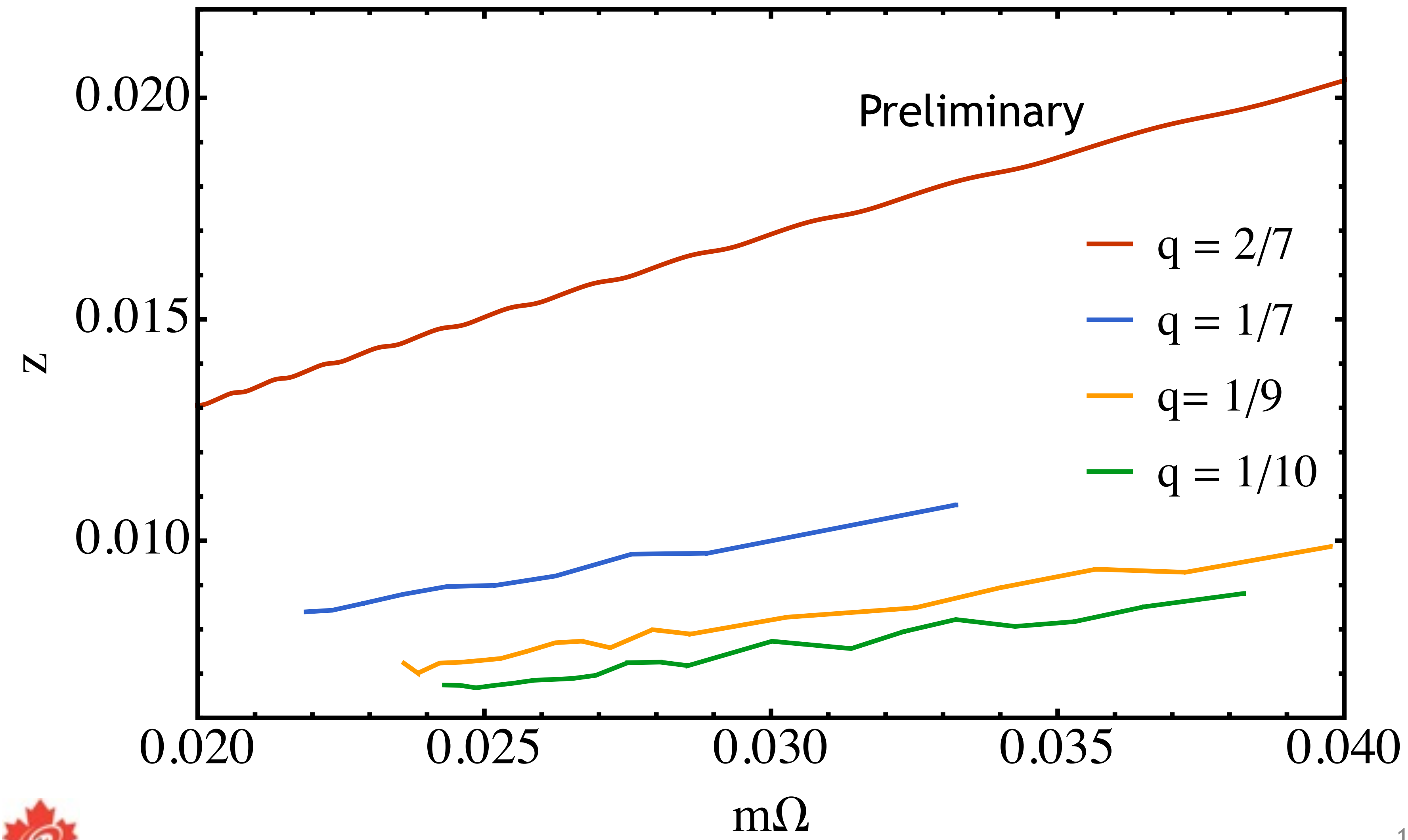
$$\Delta Q = z_1 m_1 + z_2 m_2 - (M - 2\Omega J)$$



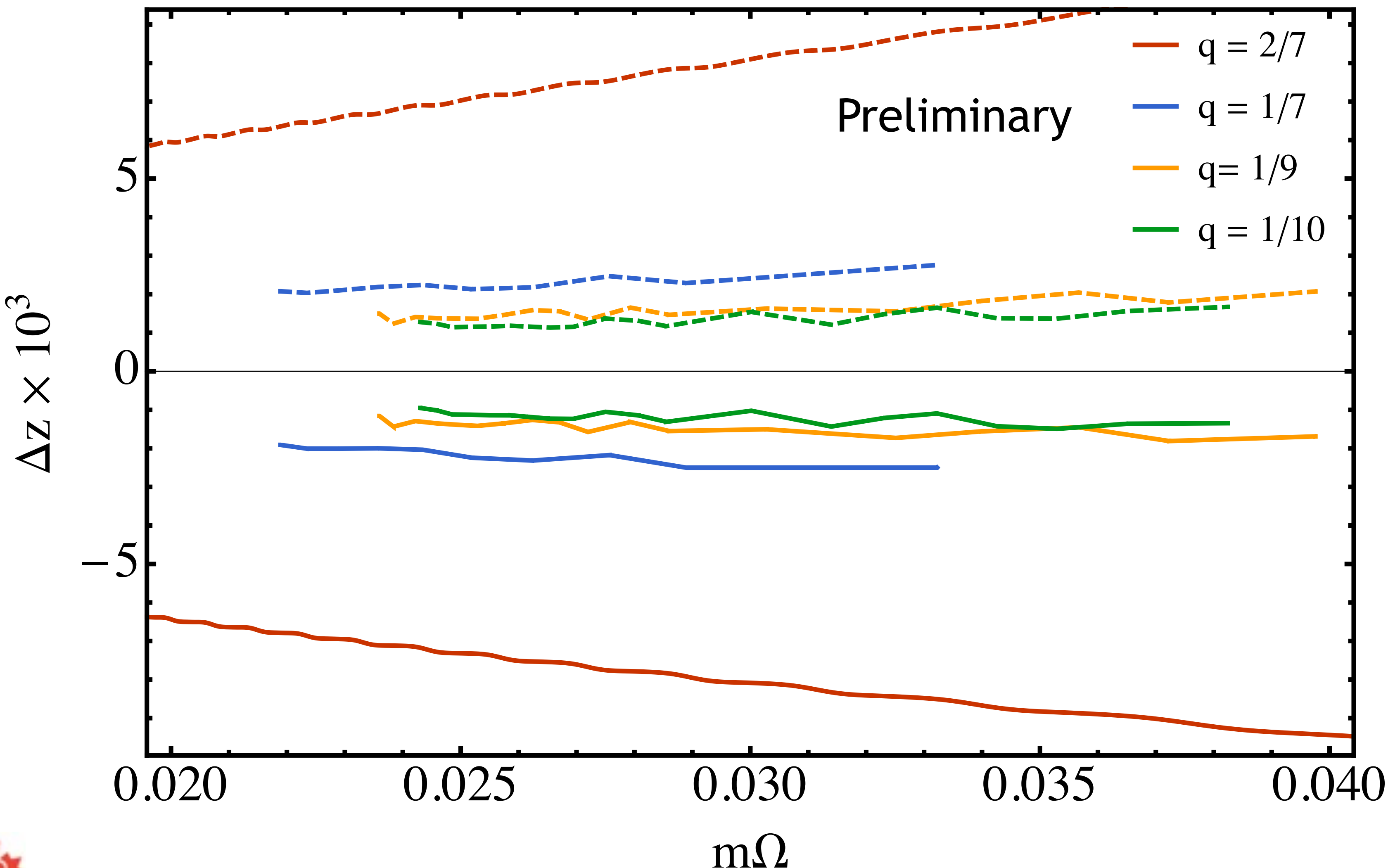
Redshift vs SF



Redshift vs SF

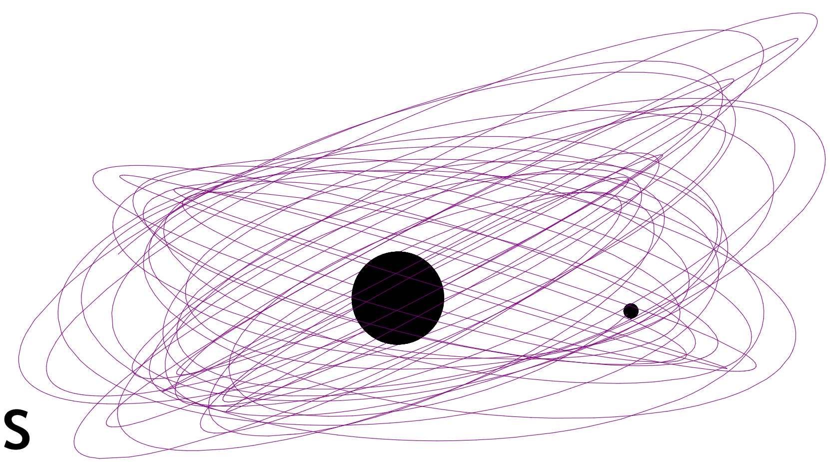


Redshift vs SF



Summary and outlook: Redshift

- Extracted redshift in NR
- Confirmed first law for binaries to 1:1000
- Higher mass ratios, high order SF
- Spinning, eccentric binaries
- Testing and extending first law of binary black holes



Outlook: SF and NR

- Already done by others: periastron advance, E(J)
- Redshift in infancy
- Self torque (hard)
- Other frequency shifts
- Tidal invariants
- Everything with spin, eccentricity
- Pushing to high mass ratios key