First order Self-force in Kerr spacetime

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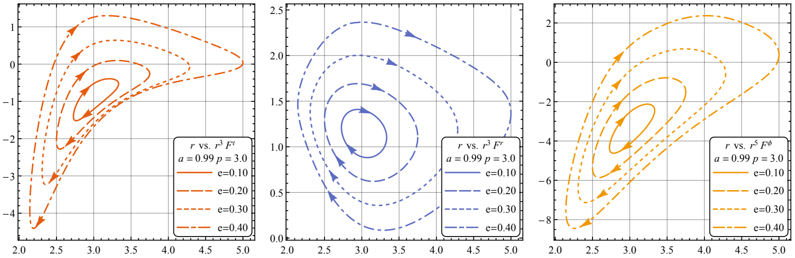
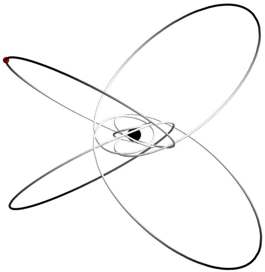
Capra 19, Meudon, 28 June 2016

arXiv:1606.06297



Outline

Introduction



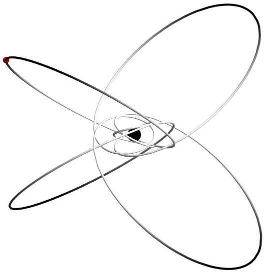
In short:

Kerr space time is not spherically symmetric!

Consequences:

- All equations are more complicated (by order of magnitude).
- Generic orbits are not planar (biperiodic, resonances).
- Linearized Einstein equation does not separate over spherical harmonics.

The trouble with Kerr



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Time domain approaches

Strategy

- Use axisymmetry to decouple "m-mode".
- Solve 2+1D PDEs numerically.
- Use "effective source" regularization scheme

Scalar self-force (Thornburg)

(see J. Thornburg's talk on Wednesday)
Eccentric equatorial orbits.

Gravitational self-force (Dolan & Barack)

Circular equatorial orbits.
(problems with gauge instabilities)

Alternative

Solve in 1+1D with coupled l-modes.

Frequency domain (scalar field)

Klein-Gordon equation

The Klein-Gordon equation for a massless scalar field in Kerr spacetime can be separated into ODEs in the frequency domain.

Scalar self-force (Warburton&Barack)

Calculation of the scalar self-force in Kerr spacetime for a particle on eccentric equatorial and inclined circular orbits has been implemented by [Warburton& Barack, 2010-2014].

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Teukolsky

Equations of motion for the (linear) Weyl curvature scalars ψ_0 and ψ_4 on Type D backgrounds decouple from the other curvature scalars, and can be solve using separation of variables in the frequency domain.

Gravitational Flux

ψ_0 and ψ_4 contain sufficient information to determine the flux of GWs to infinity and into the BH.

Numerical calculation for completely generic bound orbits implemented by [Drasco& Hughes, 2006].

Gravitational Self-force?

In fact ψ_0 and ψ_4 contain most information about a metric perturbation.

Question:
Can the GSF be calculated from ψ_0 or ψ_4?

Frequency domain (gravity)

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(Incomplete) History

Work	Details
[Barack & Ori, 2001]	Note difficulty due to irregularity of radiation gauge.
[Keidl, Friedman & Wiseman, 2007]	Metric for static particle flat space/Schwarzschild
[Keidl, Shah, Friedman, Kim & Price, 2010& 2011]	GSF and redshift for circular orbits in Schwarzschild
[Shah, Friedman & Keidl, 2012]	Redshift circular equatorial orbits Kerr
[Pound, Merlin, & Barack, 2013]	Rigorous formulation of GSF in radiation gauge
[MvdM & Shah, 2015]	Redshift for eccentric equatorial orbits in Kerr
[MvdM, 2016]	GSF for eccentric equatorial orbits in Kerr.

Method

Teukolsky equation

Teukolsky equation

$$\hat{\mathcal{T}}^{(2)} \circ \Phi_s = S$$

Teukolsky variables

ψ_0: Teukolsky variable of spin-weight +2
$\rho^{-4}\psi_4$: Teukolsky variable of spin-weight -2

Separation of variables

$$\Phi_s = \sum_{lm\omega} {}_sR_{lm\omega}(r)\, {}_sS_{lm\omega}(\theta)\, e^{i(m\phi-\omega t)}$$

${}_sS_{lm\omega}(\theta)$: spin-weighted spheroidal harmonic
${}_sR_{lm\omega}(r)$: solution of radial Teukolsky equation

$$\left(\Delta^{-s}\frac{\mathrm{d}}{\mathrm{d}r}(\Delta^{s+1}\frac{\mathrm{d}}{\mathrm{d}r}) + {}_sV_{lmw}(r)\right) {}_sR_{lm\omega} = {}_sS_{lm\omega}[T^{\mu\nu}]$$

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MST method

Series solution

$$_sR_{lm\omega}(r) = \mathcal{C} \sum_{n=-\infty}^{\infty} a_n^\nu \, F_n^\nu(r)$$

- $F_n^\nu(r)$: Hypergeometric function
- a_n^ν satisfies $\alpha_n^\nu a_{n-1}^\nu + \beta_n^\nu a_n^\nu + \gamma_n^\nu a_{n+1}^\nu = 0$
- Two independent solutions for ν give rise to independent homogeneous solutions

Advantages

- Analytic implementation of boundary conditions
- Arbitrary precision implementation possible
- Numerical implementation for flux calculations of generic orbits.[Fujita et al., 2009]

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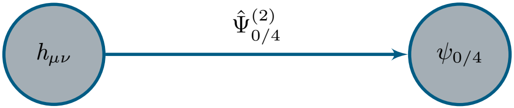
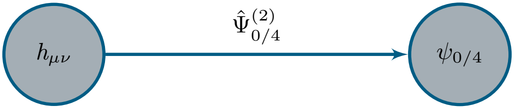


Theorem [Wald, 1974]

For any vacuum Type-D (i.e $\psi_0 = \psi_4 = 0$) background spacetime

$$\ker \hat{\Psi}^{(2)}_{0/4} = \begin{cases} \text{Gauge modes} \\ \text{Perturbations of the background with the class of vacuum type-D spacetimes (i.e. } \delta M, \delta J, \delta\alpha, \text{ or } \delta Q_{NUT}) \end{cases}$$



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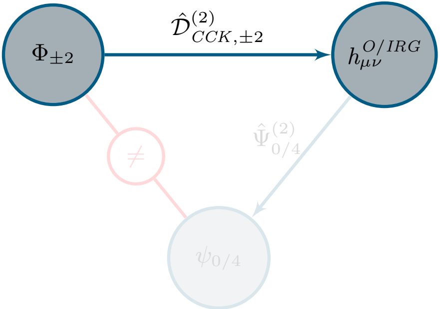
(vacuum)



IRG: $l^\mu h_{\mu\nu} = 0$

ORG: $n^\mu h_{\mu\nu} = 0$

$g^{\mu\nu} h_{\mu\nu} = 0$

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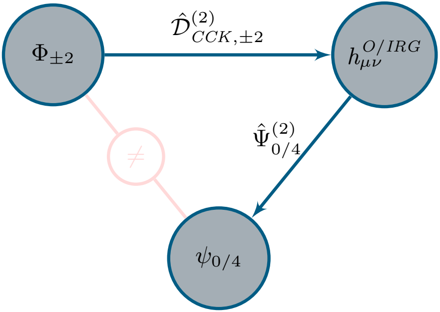

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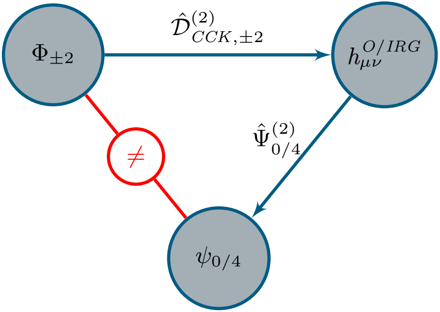

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Hertz potential

Requiring the previous diagram to commute implies four 4th order PDEs relating $\Phi_{I/ORG}$ to $\psi_{0/4}$:

$$\left(\hat{\mathcal{R}}^{(4)}_{IRG} \equiv \hat{\Psi}^{(2)}_{0} \circ \left(\hat{\mathcal{D}}^{(2)}_{CCK,-2}\right)\right) \circ \bar{\Phi}_{IRG} = \psi_0$$

$$\left(\hat{\mathcal{L}}^{(4)}_{IRG} \equiv \rho^{-4}\hat{\Psi}^{(2)}_{4} \circ \left(\hat{\mathcal{D}}^{(2)}_{CCK,-2}\right)\right) \circ \bar{\Phi}_{IRG} = \rho^{-4}\psi_4$$

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Key fact:

- $\hat{\mathcal{R}}^{(4)}_{I/ORG}$ is independent of θ.
- $\hat{\mathcal{L}}^{(4)}_{I/ORG}$ is independent of r.

Algebraic inversion of radial equation [Ori, 2001]

Radial equation

$$\hat{\mathcal{R}}^{(4)}_{ORG} \circ \bar{\Phi}_{ORG} = \rho^{-4}\psi_4$$

Mode expansions

$$\rho^{-4}\psi_4 = \sum_{lm\omega} {}_{-2}R_{lm\omega}(r)\,{}_{-2}S_{lm\omega}(\theta)\,e^{i(m\phi-\omega t)}$$

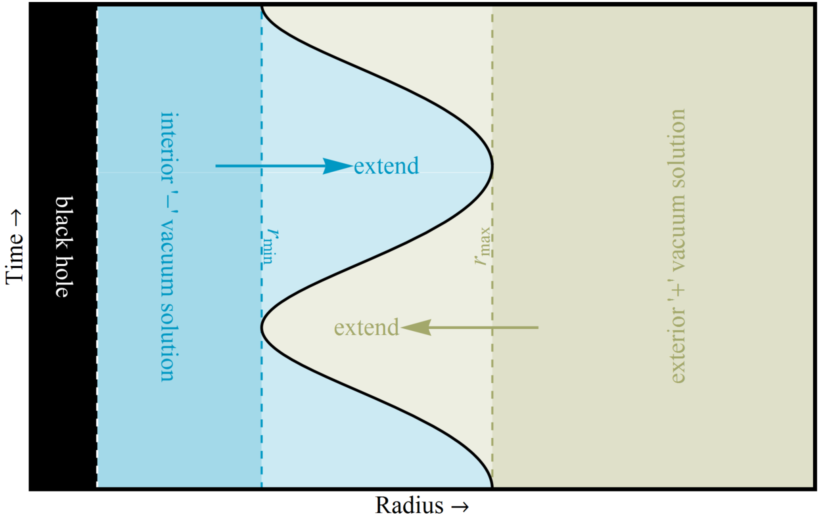
$$\Phi_{ORG} = \sum_{lm\omega} {}_{2}R_{lm\omega}(r)\,{}_{2}S_{lm\omega}(\theta)\,e^{i(m\phi-\omega t)}$$

Radial operator separates over modes

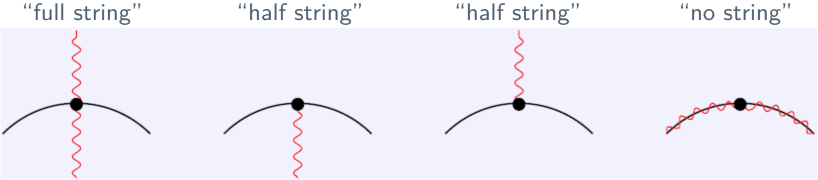
$$Y_i\left(\hat{\mathcal{R}}^{(4)}_{m\omega} \circ {}_2R^i_{lm\omega}(r)\right) = X_j\;{}_{-2}R^j_{lm\omega}(r)$$

$\hat{\mathcal{R}}^{(4)}_{m\omega}$: {hom. sol. of spin+2 radial eq.} $\to$ {hom. sol. of spin-2 radial eq.}

Method of extended homogeneous solutions



Radiation gauge metric perturbations have (gauge) string singularities.
[Barack& Ori, 2001]



Credit: C. Merlin

[Pound,Merlin&Barack, 2013]

- GSF can be calculated in “half-string” gauges, but no zero corrections to regularization parameters occur.
- Corrections to Lorenz gauge RPs cancel in “no string” gauge:

$$F^{\mu} = \Big(\sum_{l=0}^{\infty} \frac{F^{\mu,+}_{l,\mathrm{Rad}} + F^{\mu,-}_{l,\mathrm{Rad}}}{2} - B^{\mu}_{\mathrm{Lor}} - \frac{C^{\mu}_{\mathrm{Lor}}}{L} \Big) - D^{\mu}_{\mathrm{Lor}}.$$

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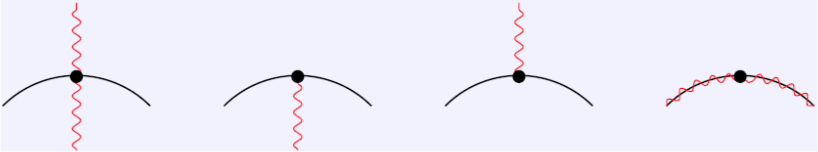

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Gravitational self-force (extension)

Compute self-force from Hertz potential:

$$
\begin{aligned}
\mathcal{F}^{\mu,\pm}_{\text{Rad}} &= \hat{\mathcal{F}}^{\mu,(1)} \circ \hat{\mathcal{D}}^{(2)}_{CCK,+2} \circ \Phi^{\pm}_{ORG} \\
&= \hat{\mathcal{F}}^{\mu,(1)} \circ \hat{\mathcal{D}}^{(2)}_{CCK,+2} \circ \sum_{m\omega\mathfrak{l}} \Psi^{\pm}_{\mathfrak{l}m\omega}\, {}_2R^{\pm}_{\mathfrak{l}m\omega}(r)\, {}_2S_{lm\omega}(\theta) e^{im\phi - i\omega t} + c.c.,
\end{aligned}
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Almost l-mode decomposition, but...

1. Wrong modes
2. "+c.c." terms
3. Coefficients depend on θ

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$$\times \left(\sum_{\substack{l+m \\ \text{even}}} \mathcal{C}^{\mu}_{m\omega si}(\cos^2\theta)\; Y_{lm}(\theta) + \cos\theta \sum_{\substack{l+m \\ \text{odd}}} \tilde{\mathcal{C}}^{\mu}_{m\omega si}(\cos^2\theta)\; Y_{lm}(\theta) \right)$$

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Completion

Missing mass and angular momentum perturbation

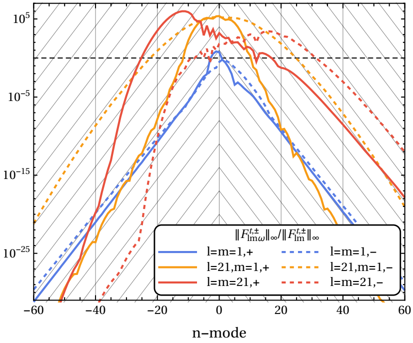
Can be found by fixing ADM mass and angular momentum and imposing smoothness of gauge invariant fields (off equator). (See Leor's talk)

Summary of method

- ψ_4 — MST method
- Φ_{ORG} — Radial inversion
- F^{μ}_{l} — Implicit choice of extension
- Mode sum — Lorenz gauge parameters, accelerate convergence by "tail fit" methods
- Completion — M and J perturbation

Tests and Results

Convergence of Fourier modes



Precision loss

- Considerable cancellations in sum over Fourier modes!
- Loss of precision grows exponentially with l.

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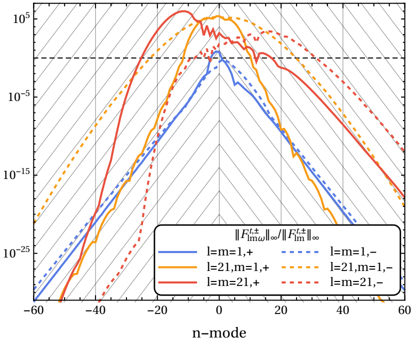

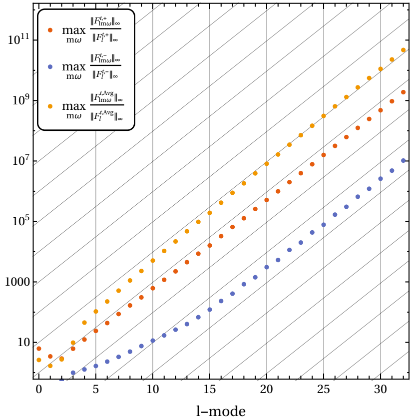

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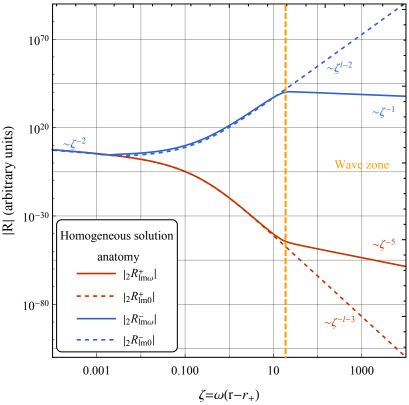


Homogeneous solution behaviour

- in far/wave zone: Mode independent powerlaw.
- in near zone: static mode behaviour.

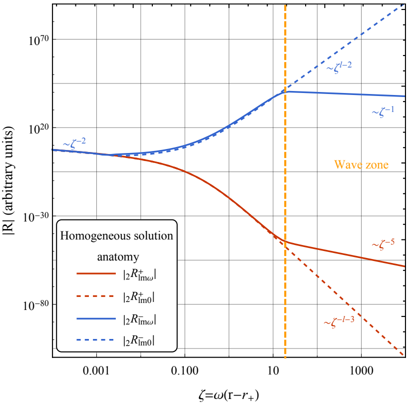
Extended homogeneous solutions

Vary $\left(\frac{r_{max}}{r_{min}}\right)^l$ over orbit.

Time domain modes

Vary $\left(\frac{r_{max}}{r_{min}}\right)^3$ over orbit.

Anatomy of homogeneous solutions



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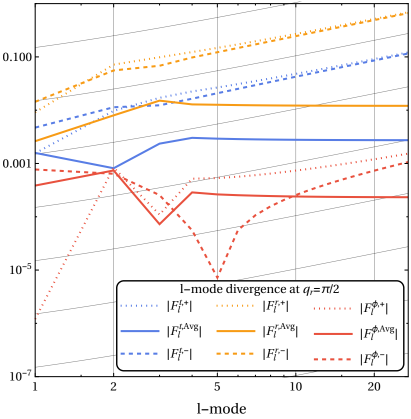
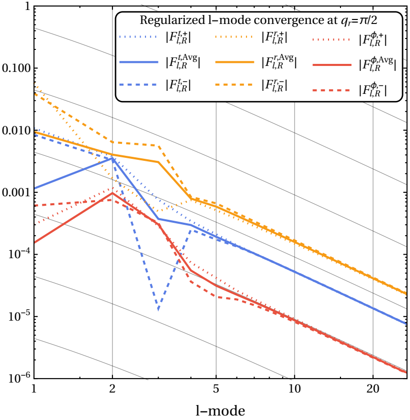
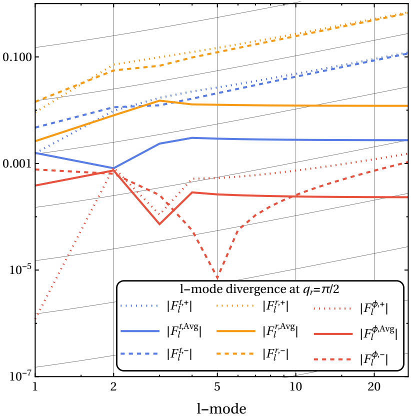
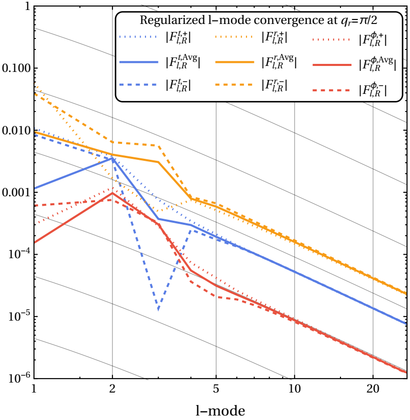
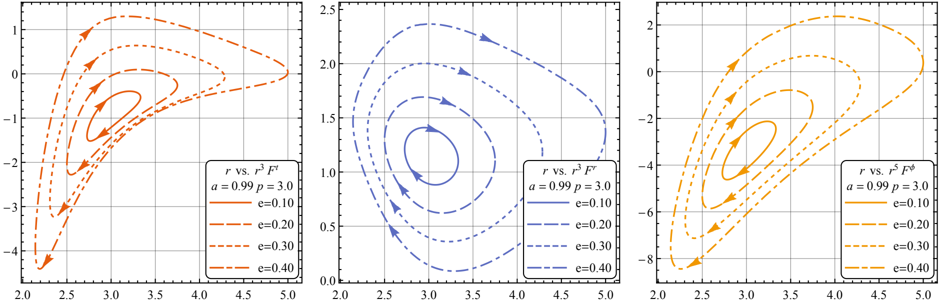

0.100
0.001
10^{-5}
10^{-7}
1
2
5
10
20
l–mode
l–mode divergence at $q_r=\pi/2$
$|F_l^{t,+}|$
$|F_l^{r,+}|$
$|F_l^{\phi,+}|$
$|F_l^{t,\mathrm{Avg}}|$
$|F_l^{r,\mathrm{Avg}}|$
$|F_l^{\phi,\mathrm{Avg}}|$
$|F_l^{t,-}|$
$|F_l^{r,-}|$
$|F_l^{\phi,-}|$


1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
1
2
5
10
20
l–mode
Regularized l–mode convergence at $q_r=\pi/2$
$|F_{l,R}^{t,+}|$
$|F_{l,R}^{r,+}|$
$|F_{l,R}^{\phi,+}|$
$|F_{l,R}^{t,\mathrm{Avg}}|$
$|F_{l,R}^{r,\mathrm{Avg}}|$
$|F_{l,R}^{\phi,\mathrm{Avg}}|$
$|F_{l,R}^{t,-}|$
$|F_{l,R}^{r,-}|$
$|F_{l,R}^{\phi,-}|$


l–mode divergence at $q_r=\pi/2$
$|F_l^{t,+}|$
$|F_l^{r,+}|$
$|F_l^{\phi,+}|$
$|F_l^{t,\mathrm{Avg}}|$
$|F_l^{r,\mathrm{Avg}}|$
$|F_l^{\phi,\mathrm{Avg}}|$
$|F_l^{t,-}|$
$|F_l^{r,-}|$
$|F_l^{\phi,-}|$
0.100
0.001
10^{-5}
10^{-7}
1
2
5
10
20
l–mode


Regularized l–mode convergence at $q_r=\pi/2$
$|F_{l,R}^{t,+}|$
$|F_{l,R}^{r,+}|$
$|F_{l,R}^{\phi,+}|$
$|F_{l,R}^{t,\mathrm{Avg}}|$
$|F_{l,R}^{r,\mathrm{Avg}}|$
$|F_{l,R}^{\phi,\mathrm{Avg}}|$
$|F_{l,R}^{t,-}|$
$|F_{l,R}^{r,-}|$
$|F_{l,R}^{\phi,-}|$
1
0.100
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
1
2
5
10
20
l–mode

Self-force loops



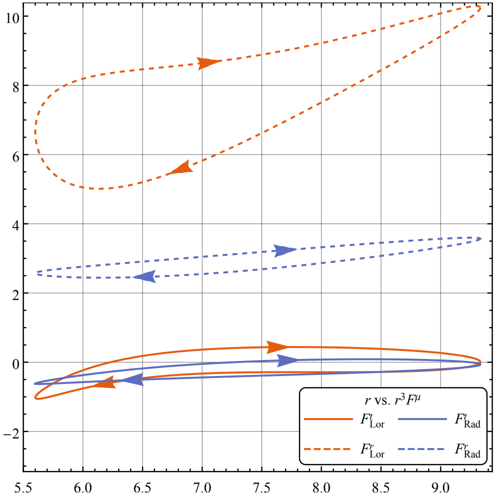
Range of capabilities

- Any value of the spin parameter a.
- Any semilatus rectum p (including fairly high whirl numbers)
- Eccentricities upto $e \lesssim 0.4$

Gauge dependence of GSF

Shown $(p, e) = (7, 0.25)$

- Radiation gauge GSF (MvdM)
- Lorenz gauge GSF (Akcay & Warburton)



Pseudo invariants and gauge completion

Pseudo-invariants

Many of the invariants we like to calculate (redshift, ISCO shift, Periapsis advance) are not true gauge invariants. Instead they are invariant under a restricted class of gauge transformations that leave frequencies unchanged. Normally only consider asymptotically flat, smooth gauges.

"No-string" gauge

The no-string gauge used for our calculating is discontinuous on a hypersurface containing the particle. Hence it is not in the desired class of gauges. It can be brought into this class by adding a suitable gauge part to the completion.

Strategy

- Only gauge transformations linear in t can effect frequencies.
- Can be constrained by requiring continuity of stationary axisymmetric part of h_{tt}, $h_{t\phi}$, and $h_{\phi\phi}$ across circular orbits.

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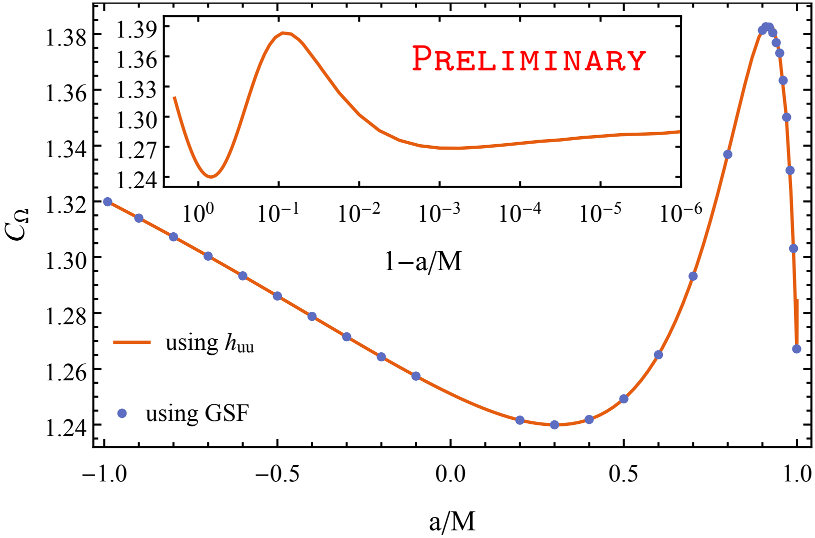
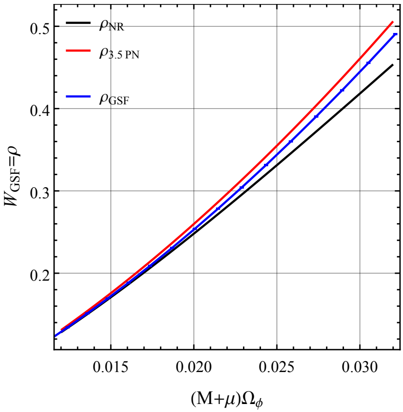
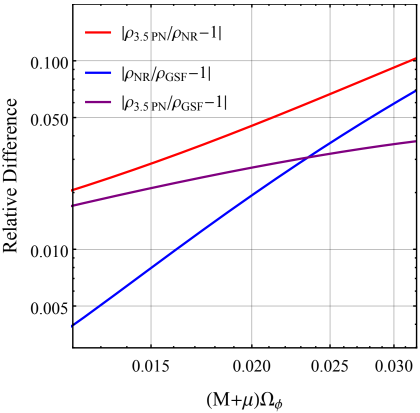
ISCO shift

$$(M+\mu)\Omega_{isco} = M\Omega^{(0)}_{isco} + \eta C_\Omega + \mathcal{O}(\eta^2)$$



$$\frac{\Omega_r^2}{\Omega_\phi^2} = W(\eta, a; x) = W_{kerr}(a; x) + \eta\rho(a; x); \quad x \equiv ((M+\mu)\Omega_\phi)^(2/3)$$





Summary and Outlook

Summary

Can calculate GSF for eccentric equatorial orbits in Kerr spacetime to moderate eccentricities.

Outlook

- Gauge completion of eccentric orbits (allows calculation of pseudoinvariants such as self-torque).
- Inclined orbits
- Inspiral evolution

The end

Thank you for listening!

For more details see

MvdM and AG Shah, arXiv:1506.04755
MvdM, arXiv:1606.06297