Highly eccentric EMRIs with self-force and spin-curvature-force

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Motivation: compact binaries

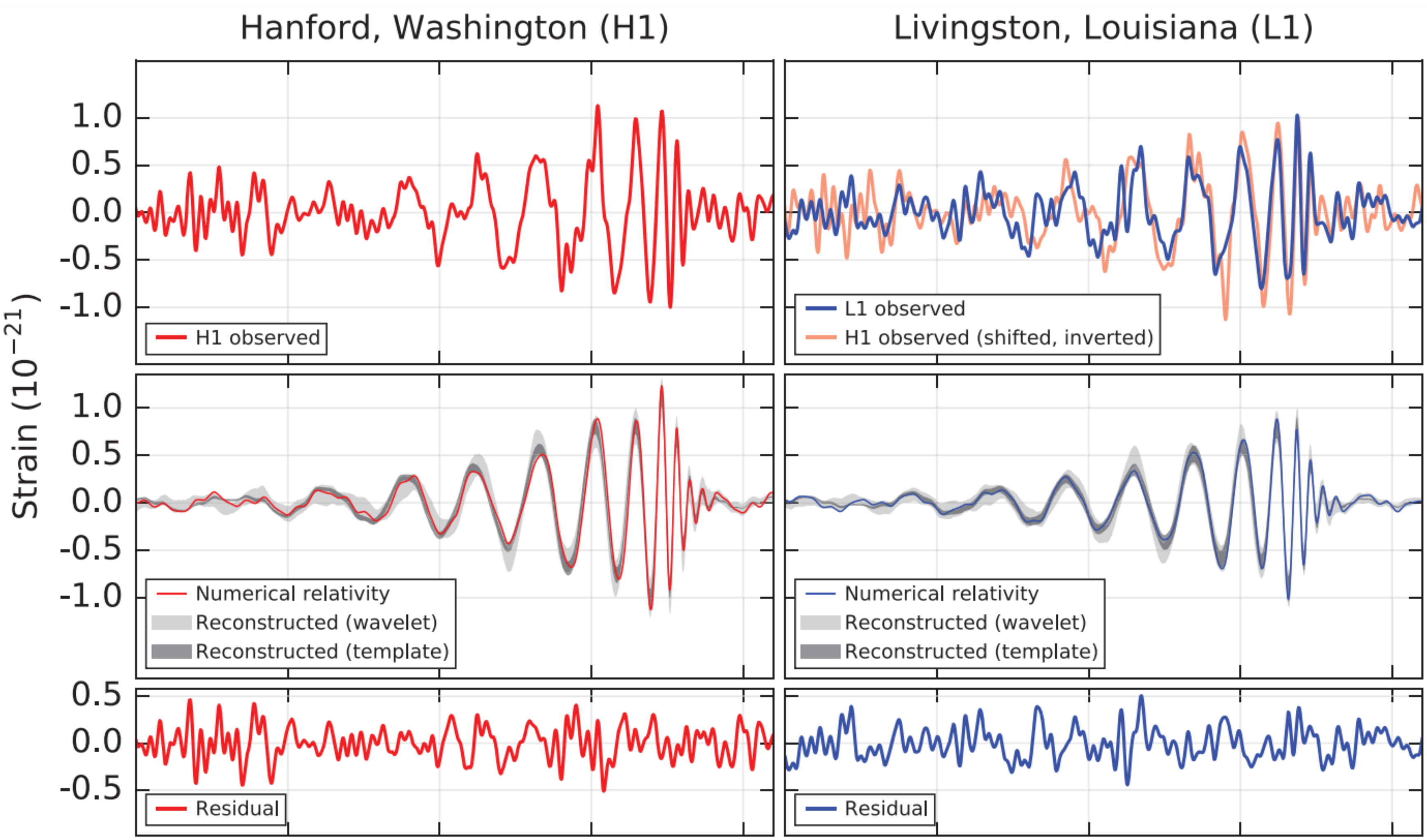
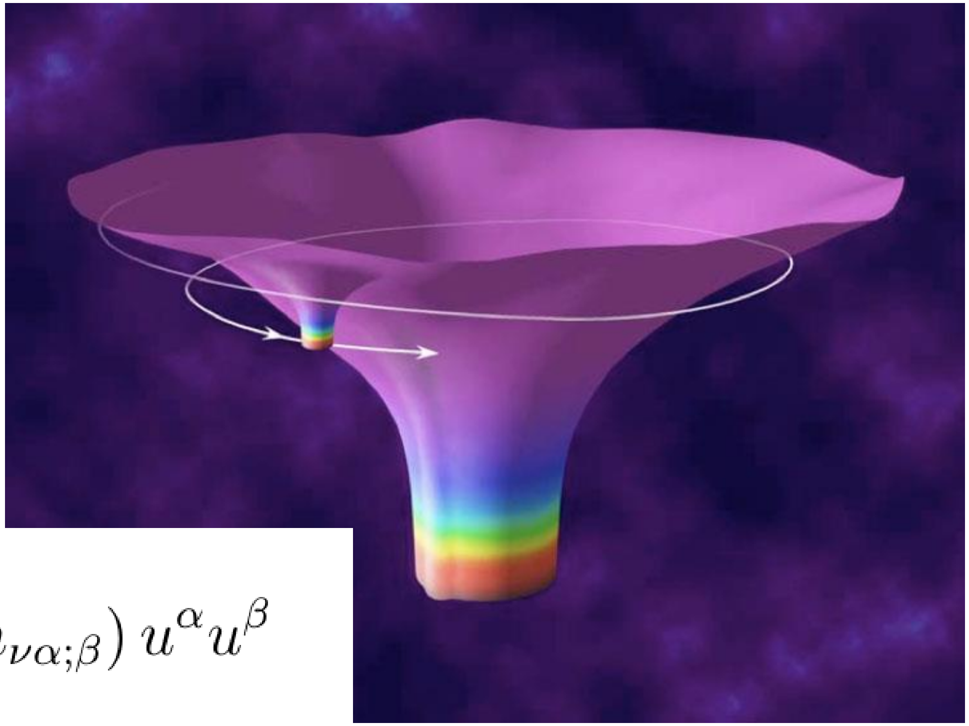


Image credit: LIGO Scientific Collaboration

Motivation: perturbation theory

- The era of gravitational wave astronomy has dawned
- Compact binaries are important sources
- Highly-relativistic small-mass-ratio binaries are not well suited for post-Newtonian or numerical relativity
- Perturb metric in powers of mass-ratio (μ/M)
- Correct motion with perturbed metric (gravitational self-force)



$$F^{\mu} = \frac{\mu}{2}\left(g^{\mu\nu} + u^{\mu}u^{\nu}\right)\left(h_{\alpha\beta;\nu} - 2h_{\nu\alpha;\beta}\right)u^{\alpha}u^{\beta}$$

Features of EMRI model used here

- First-order self-force (dissipative and conservative)
- Spin-curvature interaction (spin-force)
- Accurate: 7+ digits of force accuracy (track phase to within ~0.1 radians)
- Broad range of orbital parameters (high eccentricity)

Other important effects:

- Kerr spacetime (van de Meent, arxiv:1606.06297)
- Second-order self-force (Pound, arxiv:1510.05172)

Motivation: high eccentricity and eLISA

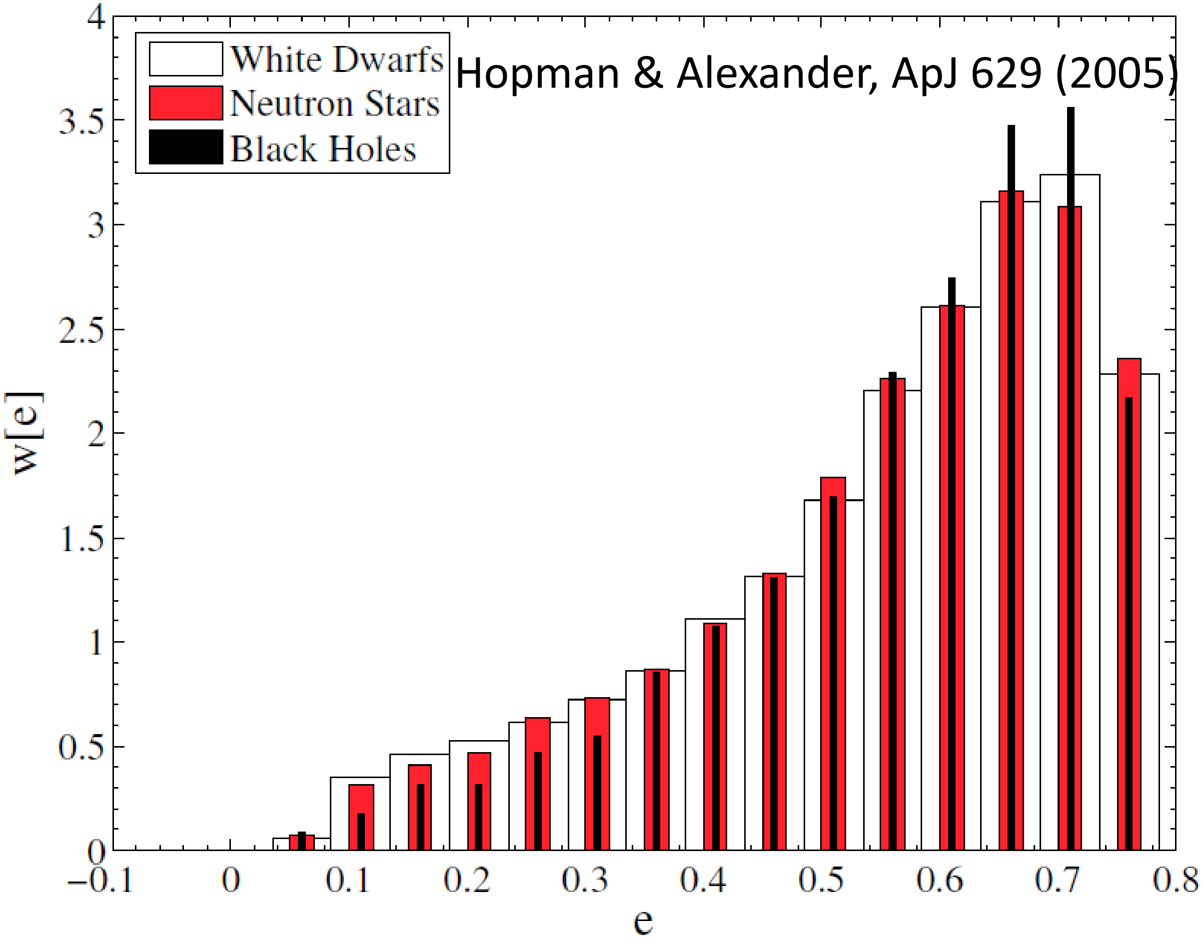
- Objects enter LISA passband with eccentricities up to e≈0.8
- Past gravitational self-force codes limited to e≲ 0.4

Warburton et al. Phys. Rev. D 85 (2012)

Akcay et al.

Phys. Rev. D 88 (2013)

Challenge: improve eccentricity range



Numerical tool: metric perturbations

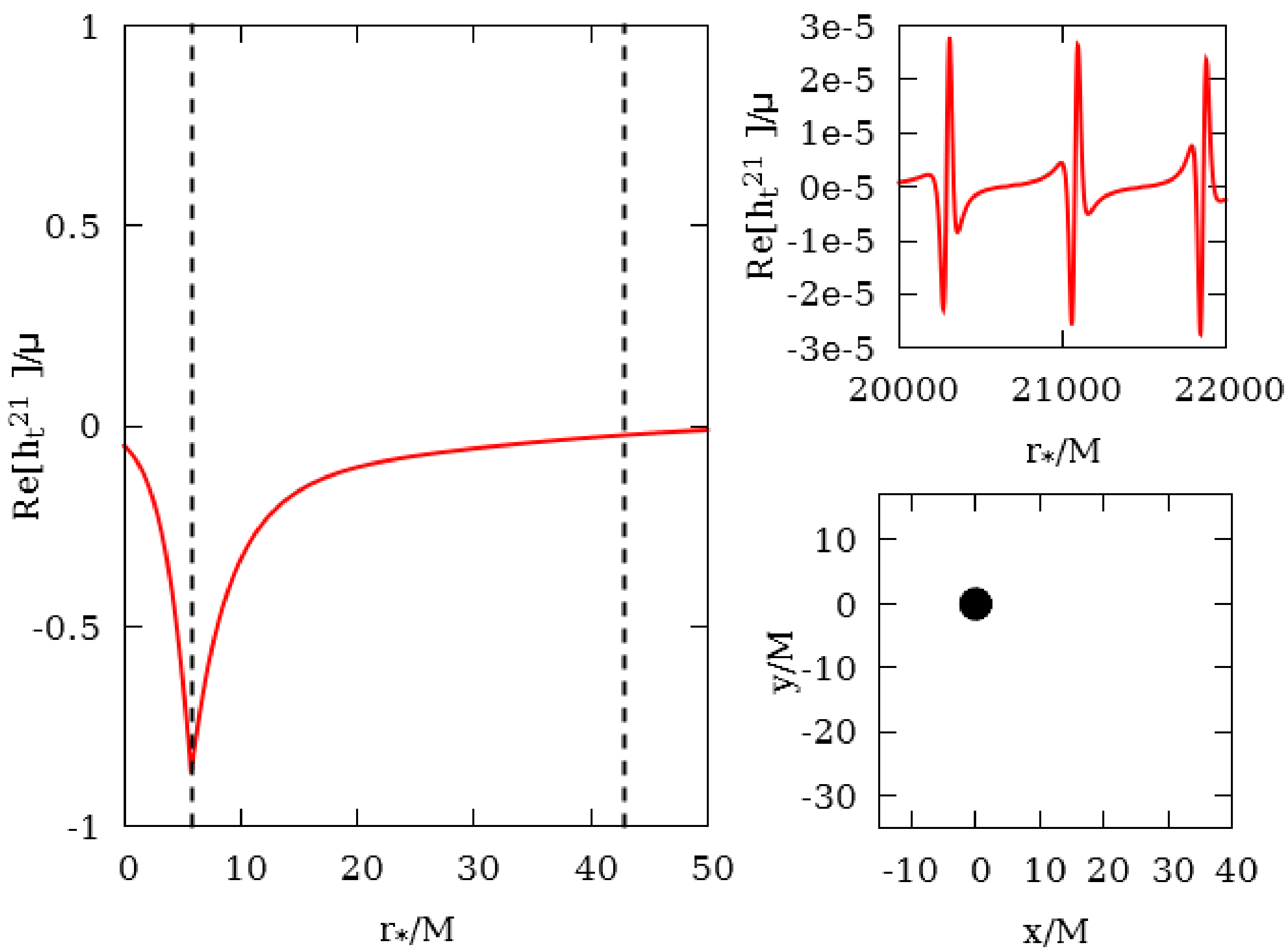
- Lorenz gauge: $\Box \bar{h}_{\mu\nu} + 2R^{\alpha\ \beta}_{\ \mu\ \nu}\bar{h}_{\alpha\beta} = -16\pi T_{\mu\nu}$
- Schwarzschild metric perturbations separable into tensor spherical harmonic and Fourier modes (l,m,n)

$$h_{\mu\nu}(t,r,\theta,\phi) = \sum_{l,m,n,k} \tilde{h}^{(k)}_{lmn}(r)\, e^{-i\omega_{mn}t}\, S^{lm(k)}_{\mu\nu}(\theta)\, e^{im\phi}$$

- Solve up to ~30,000 ODE systems (l,m,n) per orbit
- Eccentricity and separation range limited by ill-conditioning problem and computational cost
- New code developed to handle these problems

Osburn, Forseth, Evans, and Hopper, Phys. Rev. D 90 (2014); **1409.4419**

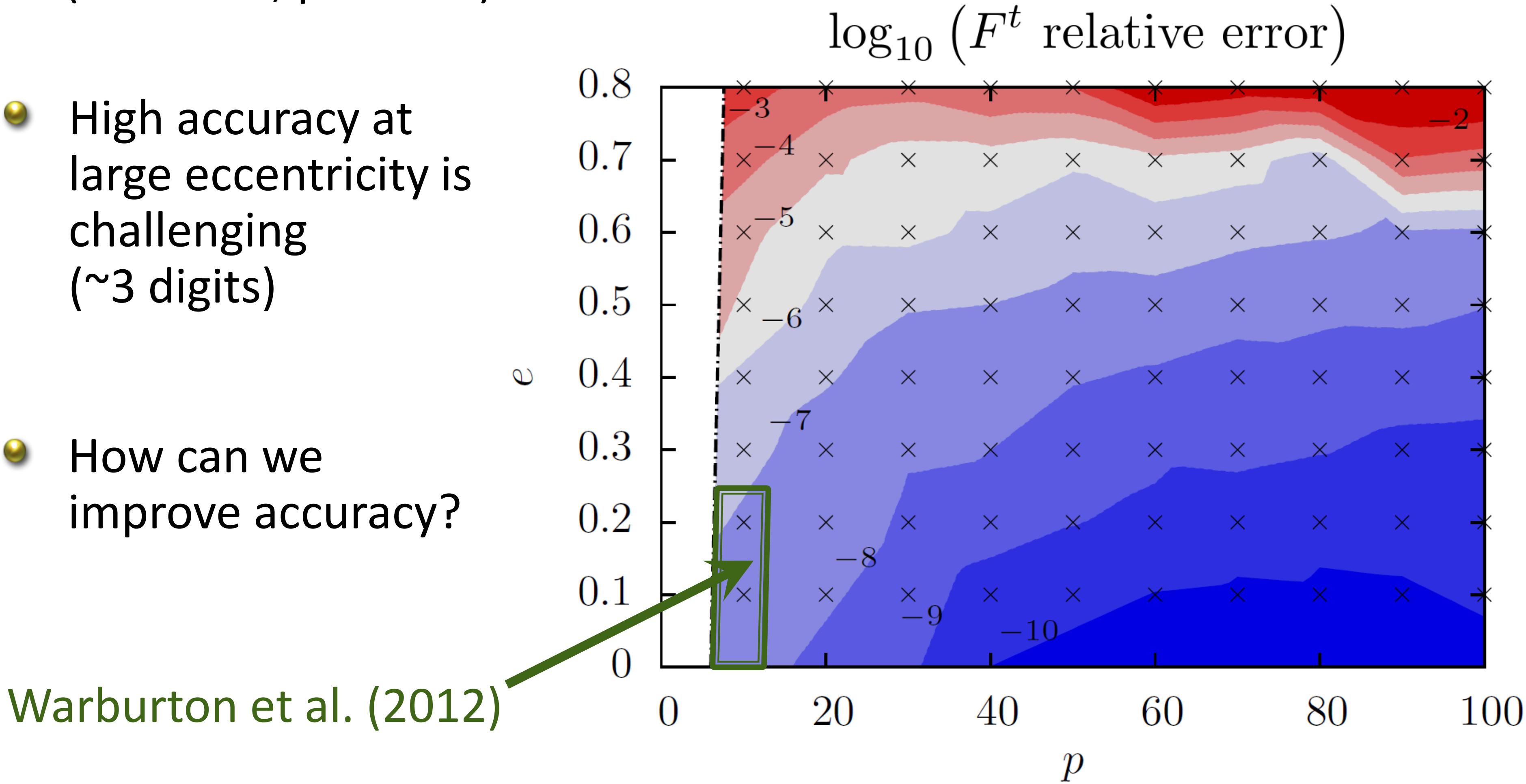
Metric perturbations and self-force



- Self-force: mode-sum regularization $F^{\mu} = \sum_{\ell'} \left[F^{\mu\,\ell'}_{\rm ret} - (2\ell'+1)A - B - \ldots \right]$

Larger domain, accuracy limitations

- We have extended the available domain of orbital parameters (e <= 0.82, p <= 100)
- High accuracy at large eccentricity is challenging (~3 digits)
- How can we improve accuracy?



Hybrid method: higher accuracy

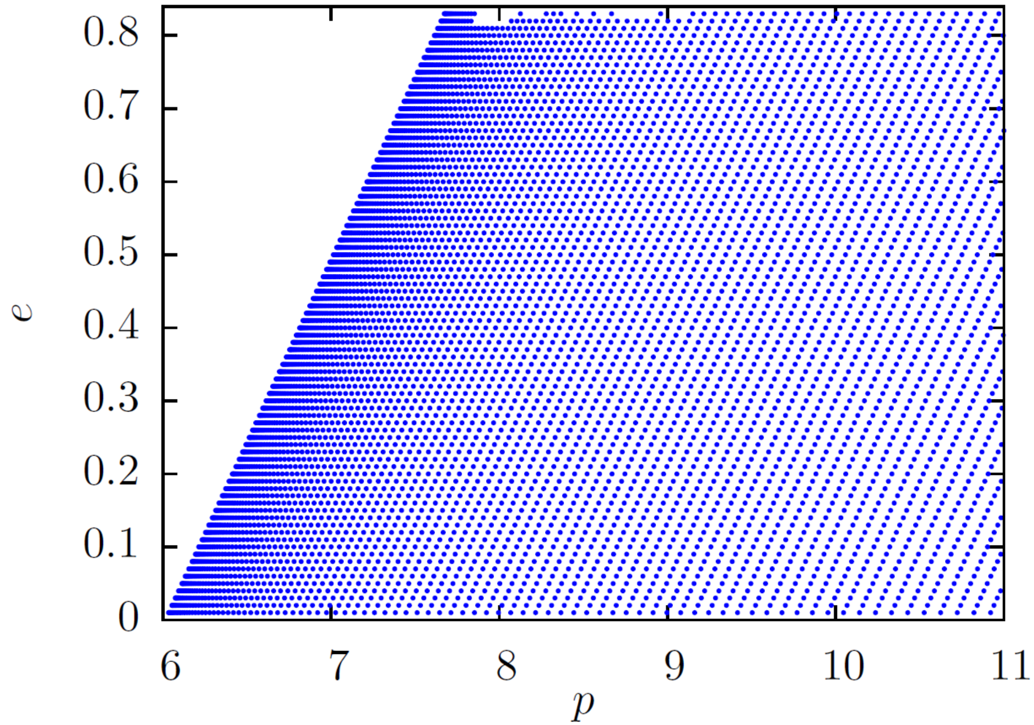
Total accumulated orbital phase: $\Phi = \kappa_0 \left(\frac{\mu}{M}\right)^{-1} + \kappa_1 + \kappa_2 \left(\frac{\mu}{M}\right) + \cdots$

$(\mu/M = 10^{-5})$ adiabatic $\approx 10^6$ rad post-1-adiabatic ≈ 10 rad

- Goal: compute orbital phase to within ~0.1 radians
- Requires self-force accuracy ≤ $(10^{-2}\mu/M) \approx 10^{-7}$
- Very hard to achieve 7+ digits at high eccentricity
- Hybrid method: Use high accuracy flux for adiabatic correction (secular approx.), GSF for post-1-adiabatic
- Carefully replace orbit averaged self-force with flux values computed in RWZ gauge

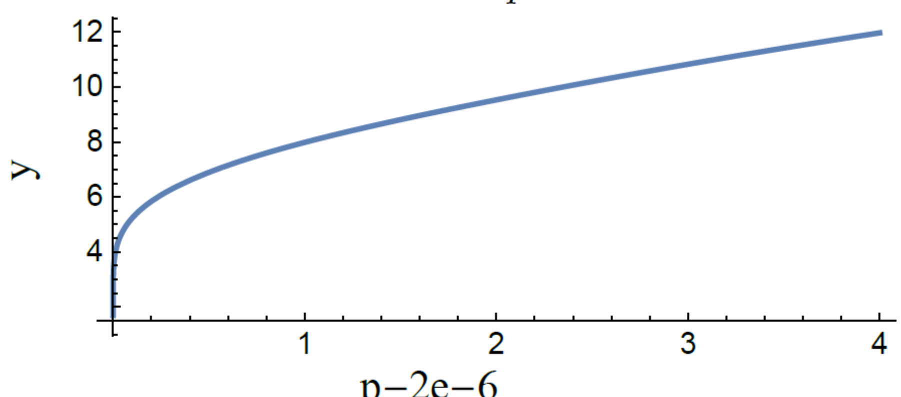
Osburn, Forseth, Evans, and Hopper, Phys. Rev. D 90 (2014); **1409.4419**

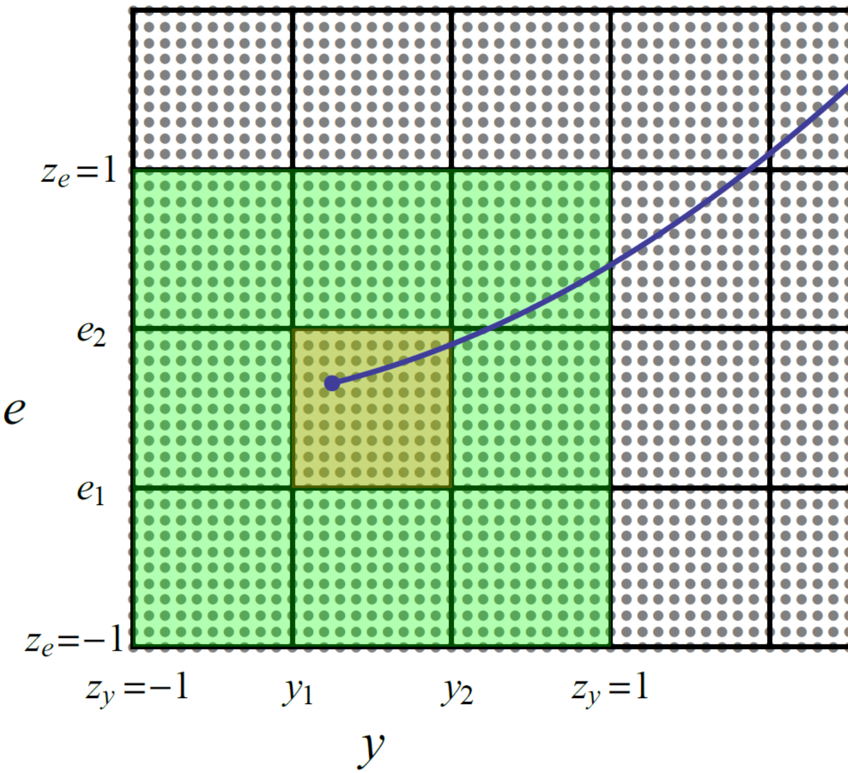
Accurate local interpolation



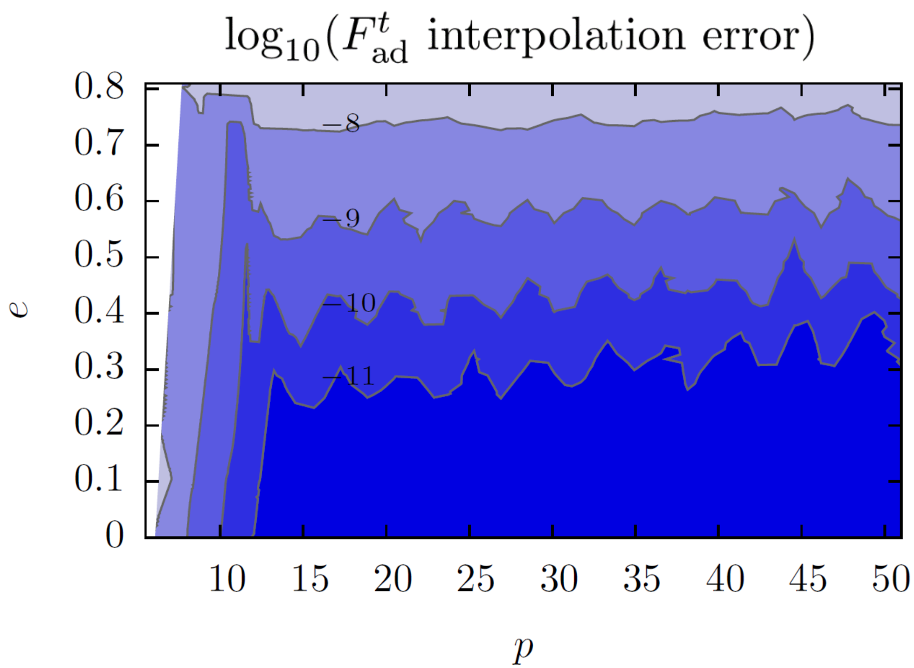
$$F^{\alpha} = \sum_{n=0}^{n_{\max}} \left[a_n^{\alpha}(e,y)\cos(nv) + b_n^{\alpha}(e,y)\sin(nv)\right]$$

$$a_n^{\alpha} = \sum_{i=0}^{i_{\max}} \sum_{j=0}^{j_{\max}} \sigma_{nij}^{\alpha} T_i(z_e) T_j(z_y)$$

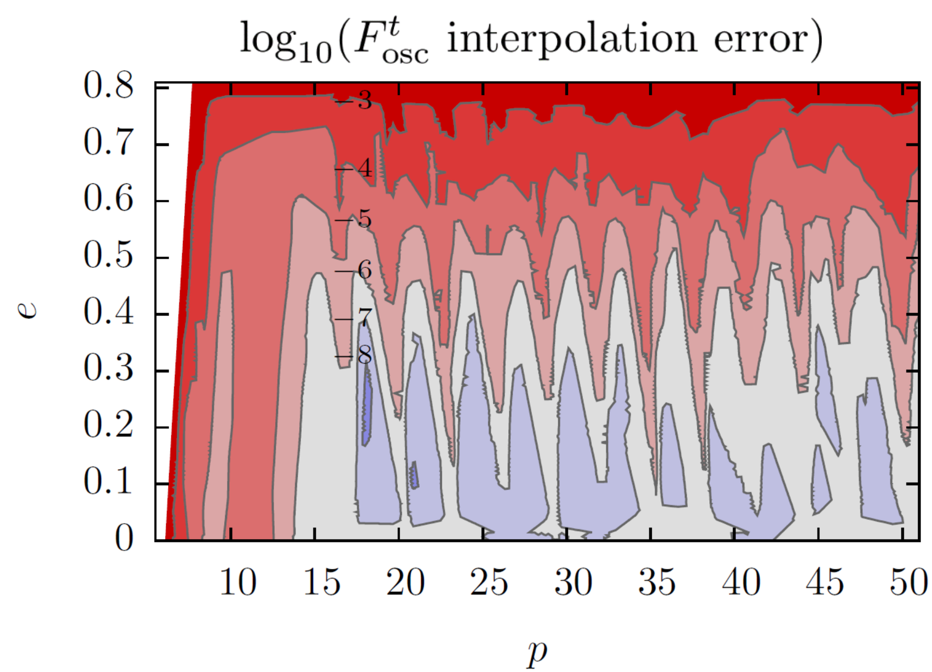




Interpolation error



- Adiabatic part calculated from accurate RWZ gauge fluxes
- Interpolate with data from 43875 orbits (2054 CPU hours)

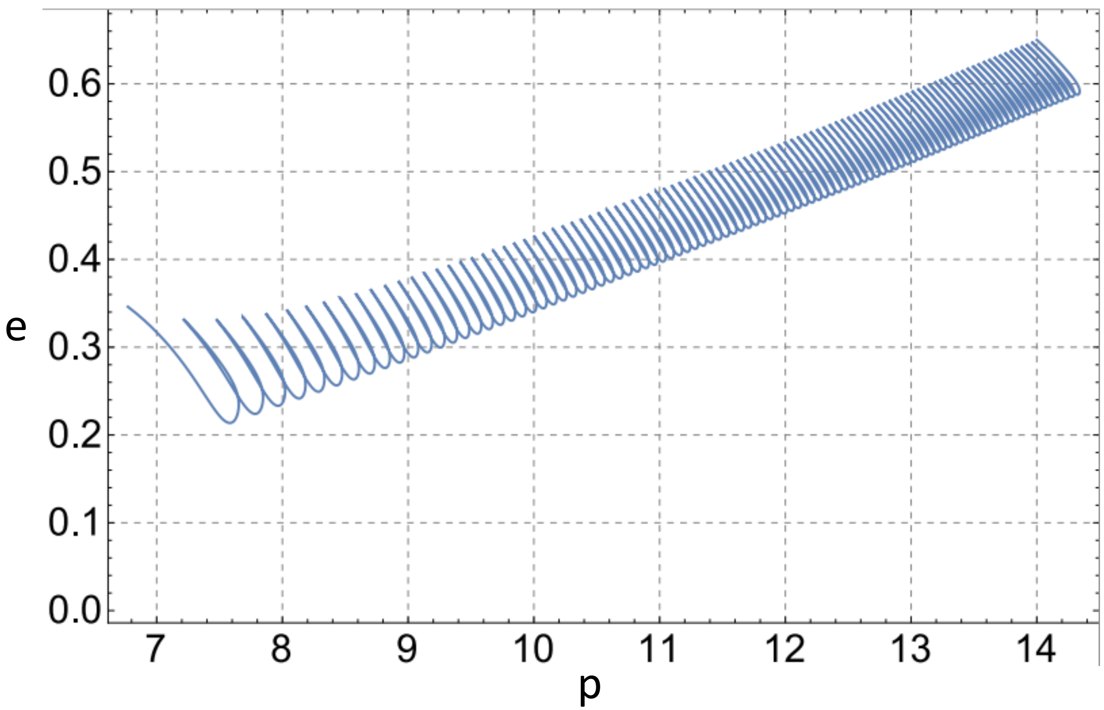


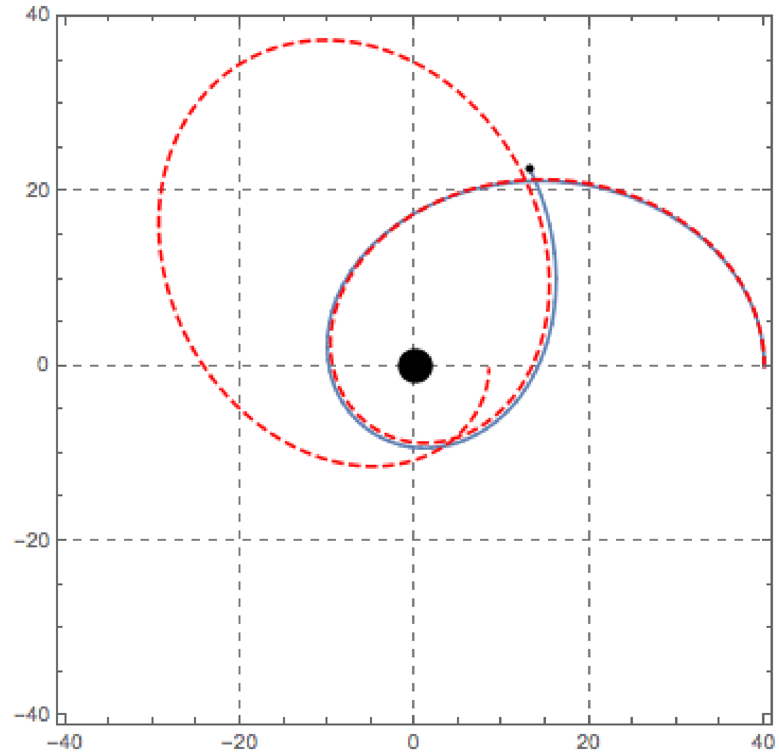
- Post-1-adiabatic part calculated from Lorenz gauge self-force
- Interpolate with data from 9602 orbits (2308 CPU hours)

Inspirals: osculating elements

- Solve ODE system for orbital parameters as functions of time

Pound and Poisson Phys. Rev. D 77 (2008)



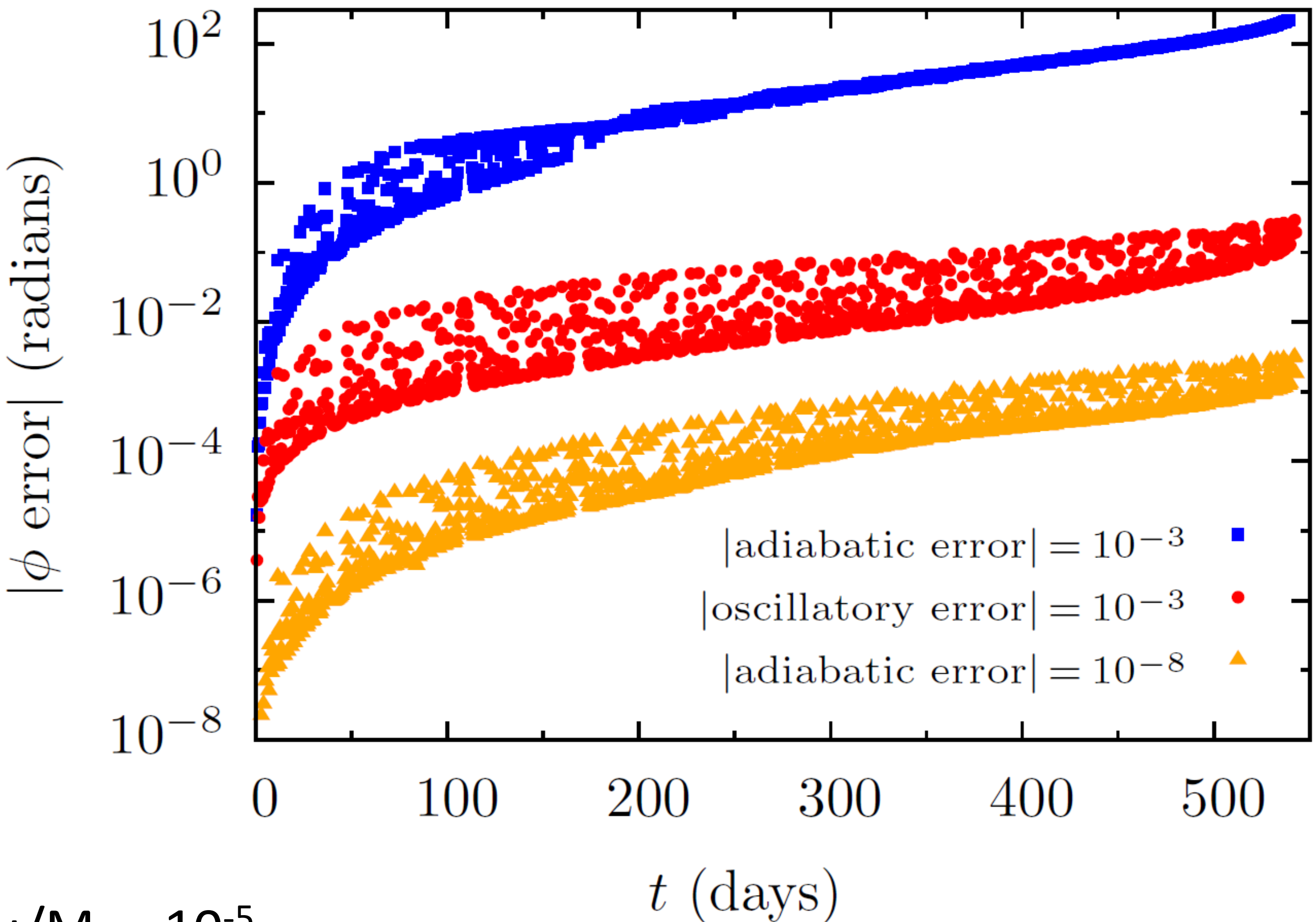


Osburn, Warburton, and Evans, Phys. Rev. D 93 (2016)

$$\frac{\mu}{M} = \frac{1}{64}$$

1409.4419

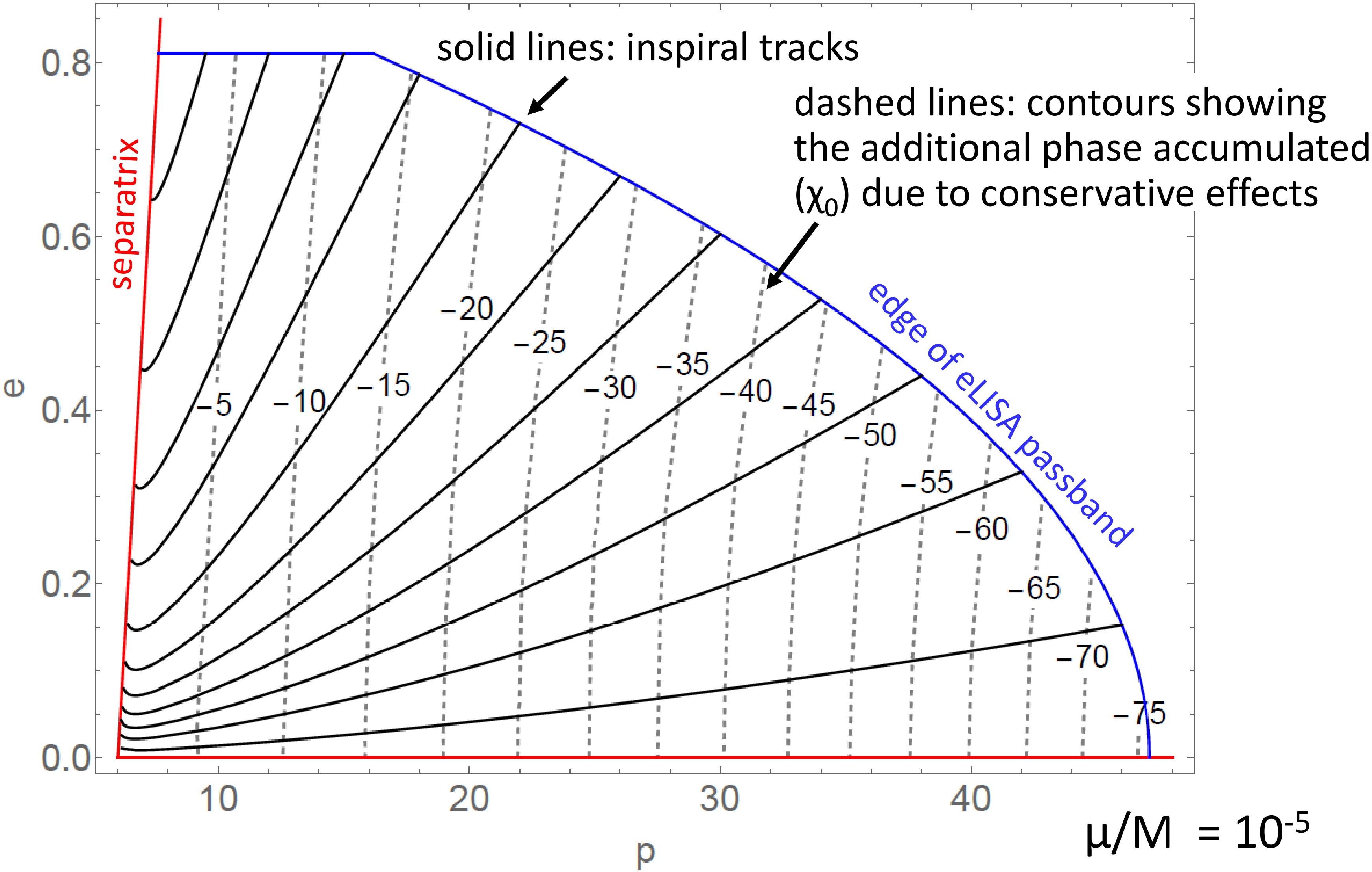
Sensitivity test of hybrid self-force



$\mu/M = 10^{-5}$

$M = 10^6 M_\odot$

Importance of conservative effects



$\mu/M = 10^{-5}$

Spin-curvature force

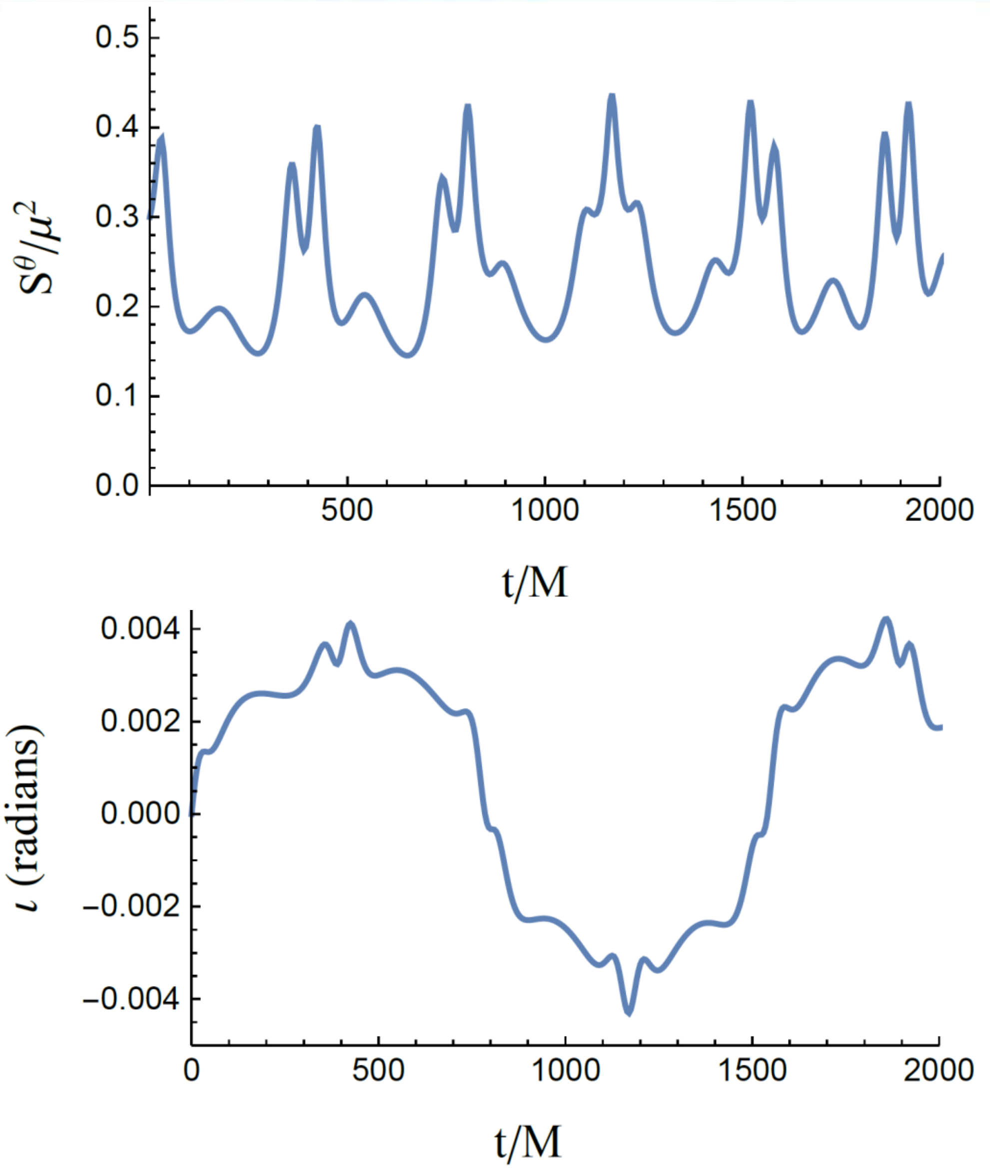
- Geodetic spin precession:

$$u^\alpha \nabla_\alpha S^\beta = 0$$

- Mathisson-Papapetrou spin-force:

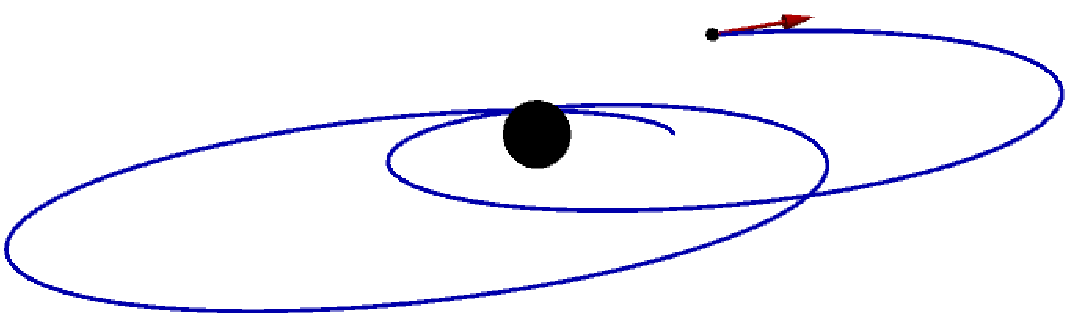
$$F^\mu_{\text{spin}} = -\frac{1}{2} R^\mu{}_{\nu\lambda\sigma} u^\nu S^{\lambda\sigma}$$

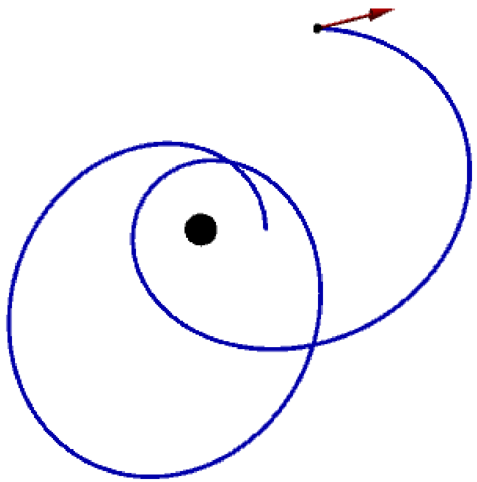
- F^θ introduced, causes orbital plane to precess
- Evolve inclination angle ι and longitude of ascending node Ω with orbital elements



Inspiral with spin-force

$$\frac{\mu}{M} = 0.04$$





Effect of aligned spin on phase

$$\frac{\mu}{M} = 0.005$$

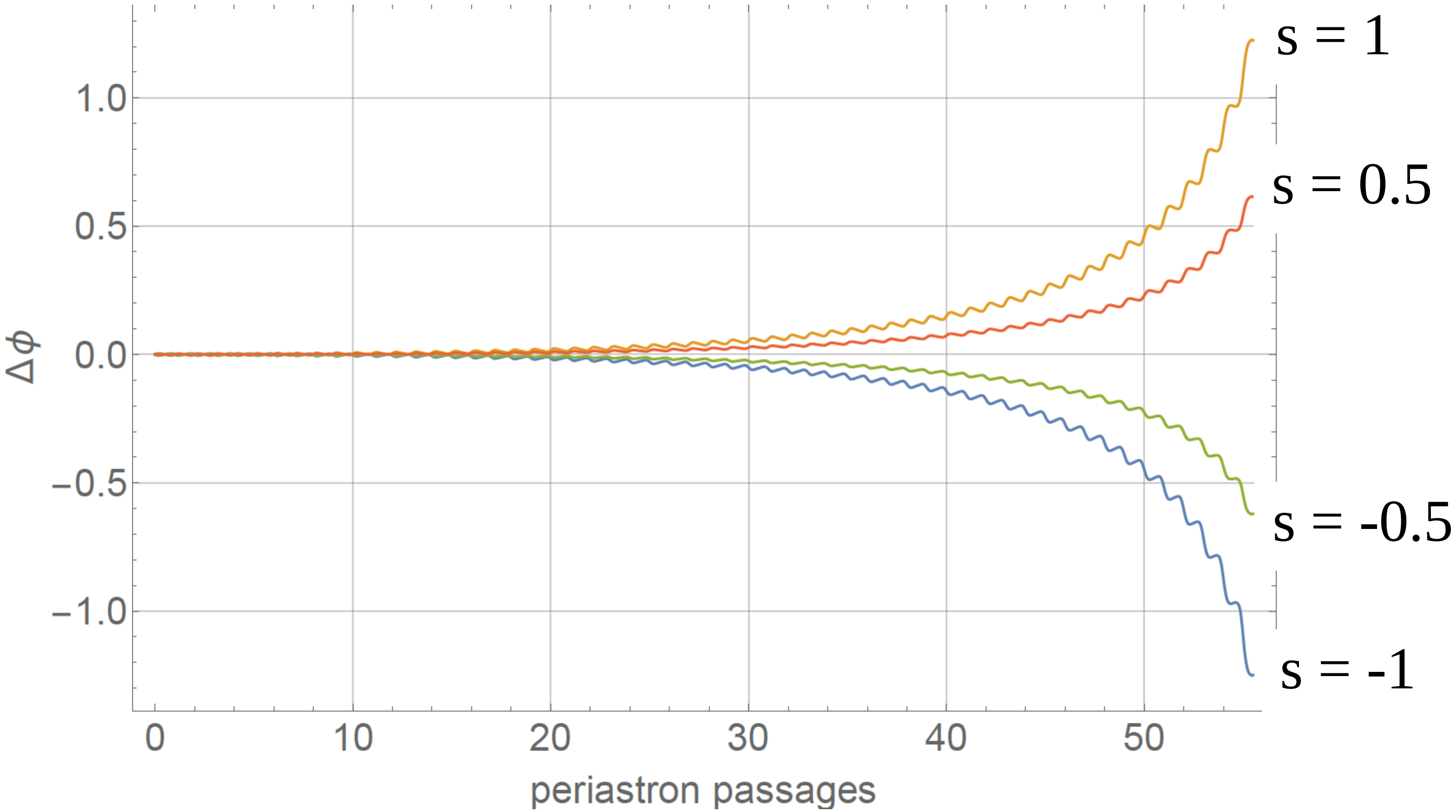
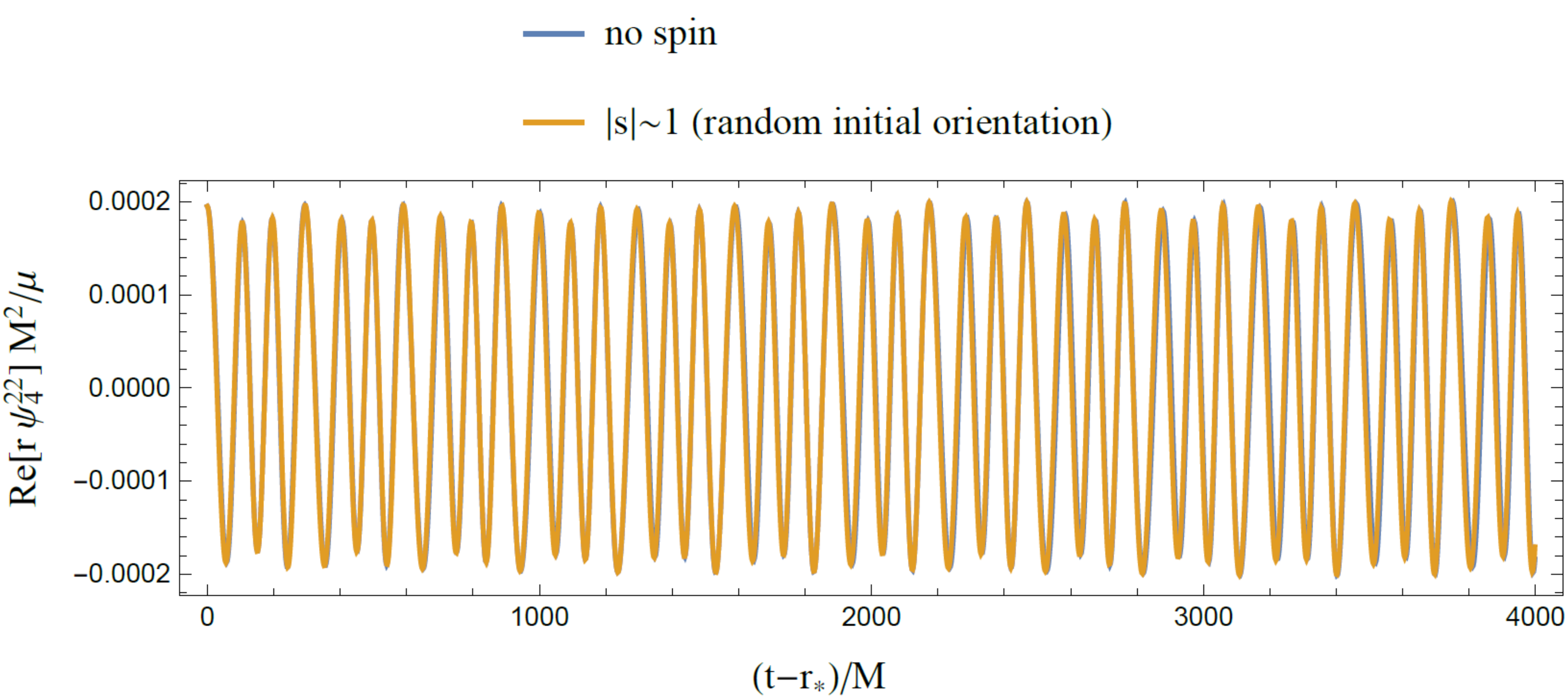


Image credit: Niels Warburton

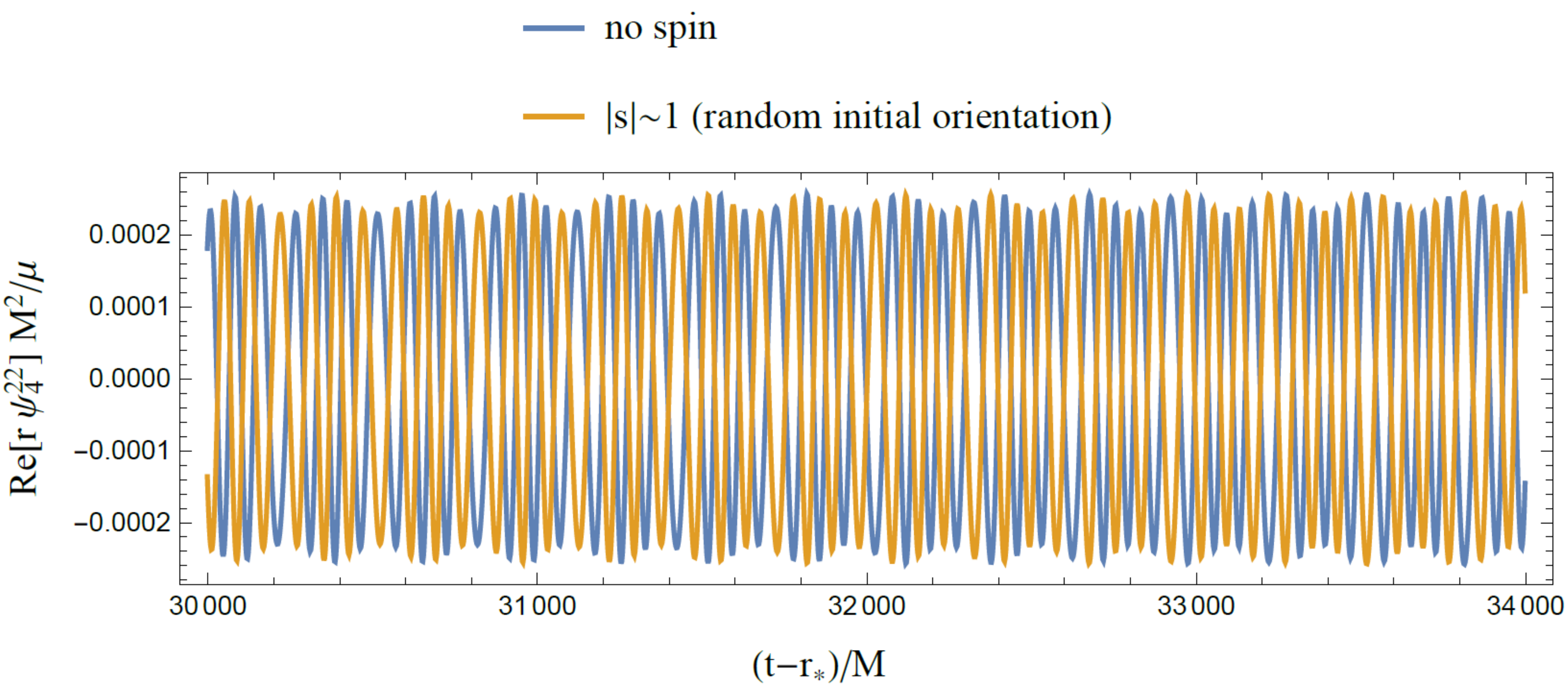
Effect of arbitrary spin on waveforms



$\frac{\mu}{M} = 0.001$ $e = 0.01$

(preliminary)

Effect of arbitrary spin on waveforms

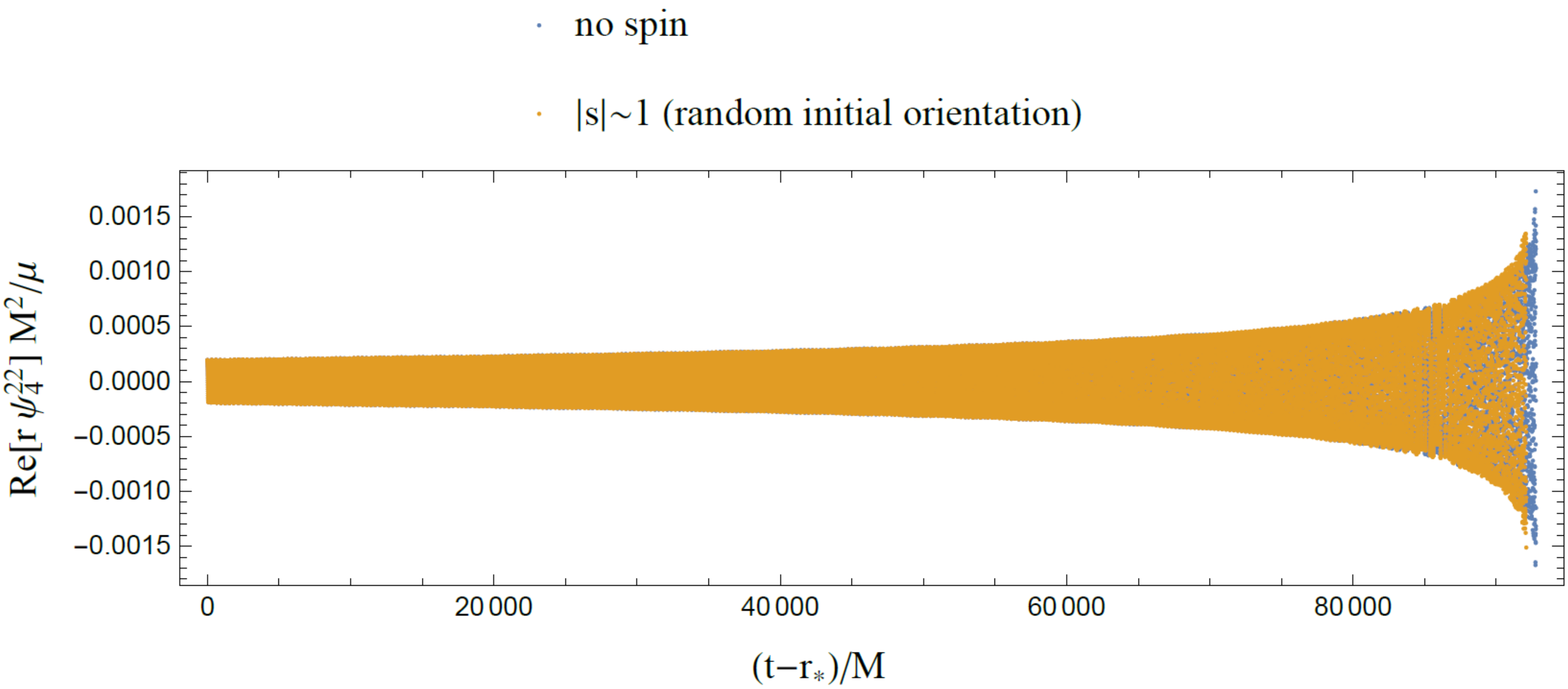


$\frac{\mu}{M} = 0.001$

e = 0.01

(preliminary)

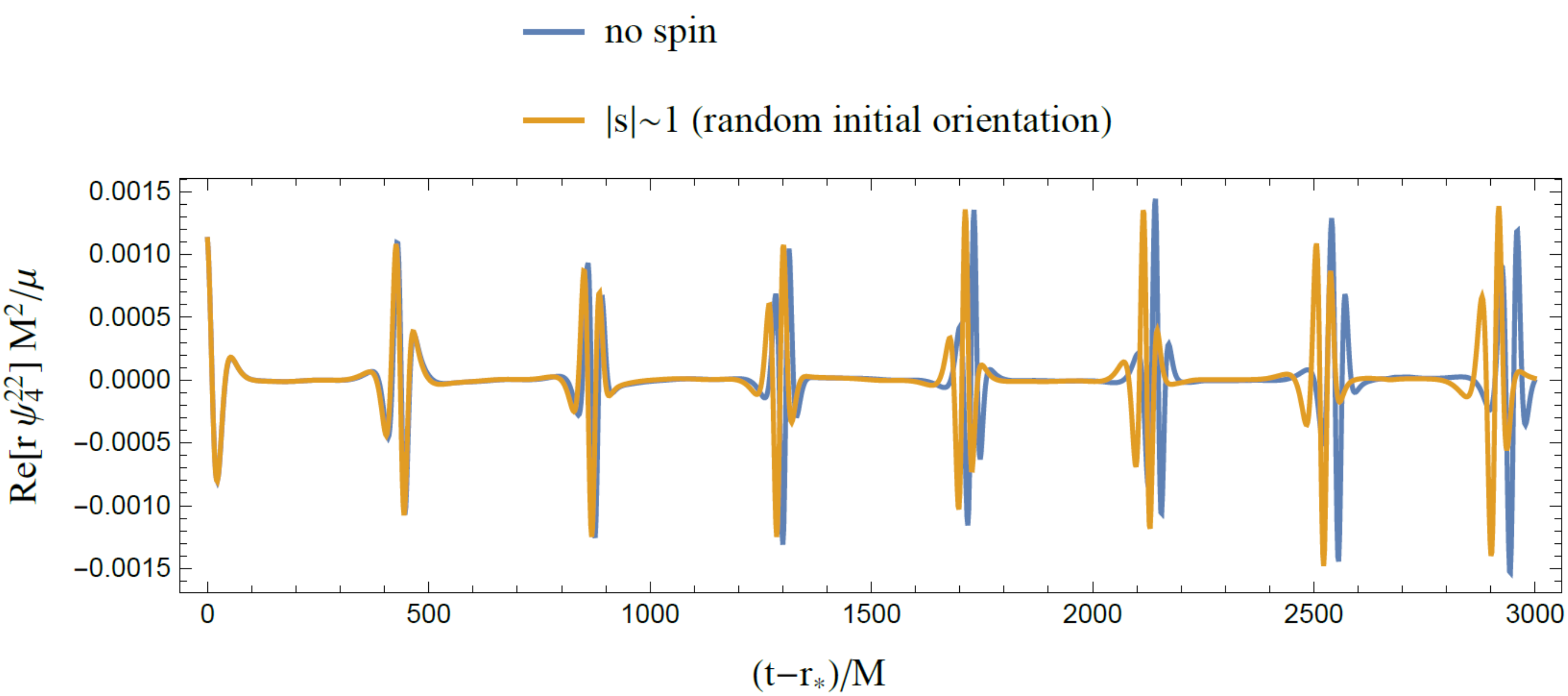
Effect of arbitrary spin on waveforms



$\frac{\mu}{M} = 0.001$ $\qquad$ $e = 0.01$

(preliminary)

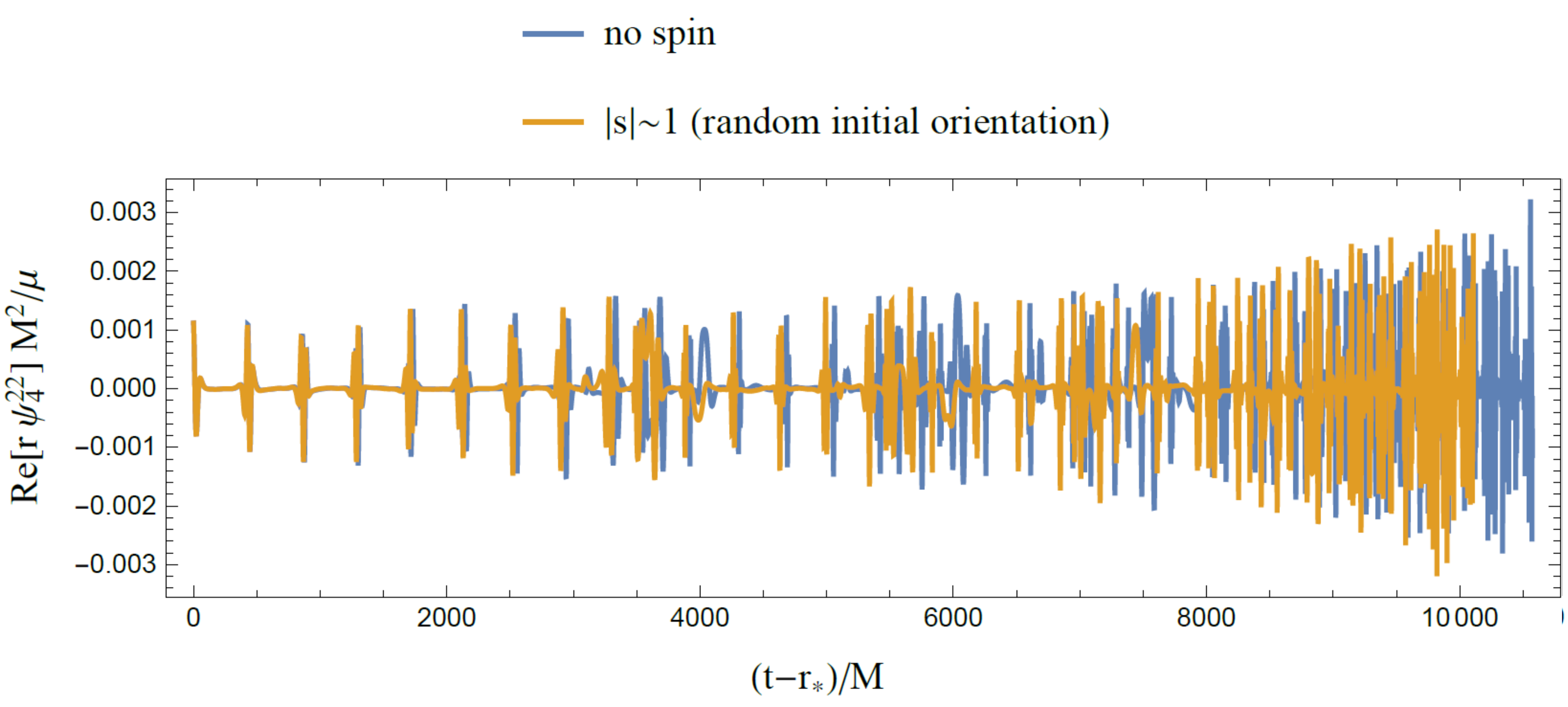
Effect of arbitrary spin on waveforms



$\frac{\mu}{M} = 0.01$ $\qquad$ e = 0.5

(preliminary)

Effect of arbitrary spin on waveforms



$\frac{\mu}{M} = 0.01$ e = 0.5

(preliminary)

Conclusions and future work

- Important problems for gravitational wave astronomy:
  - Extreme/intermediate mass ratio binaries
  - High eccentricity
  - High accuracy (7+ digits)
  - Spin-curvature coupling
- We accomplish this with the following tools:
  - Hybrid (accurate fluxes for adiabatic) self-force code
  - Add module for spin-curvature coupling (spin-force)
  - Osculating elements code generalized for inclined orbits
- Future work:
  - Kerr background
  - Second order perturbation theory

Thank you!

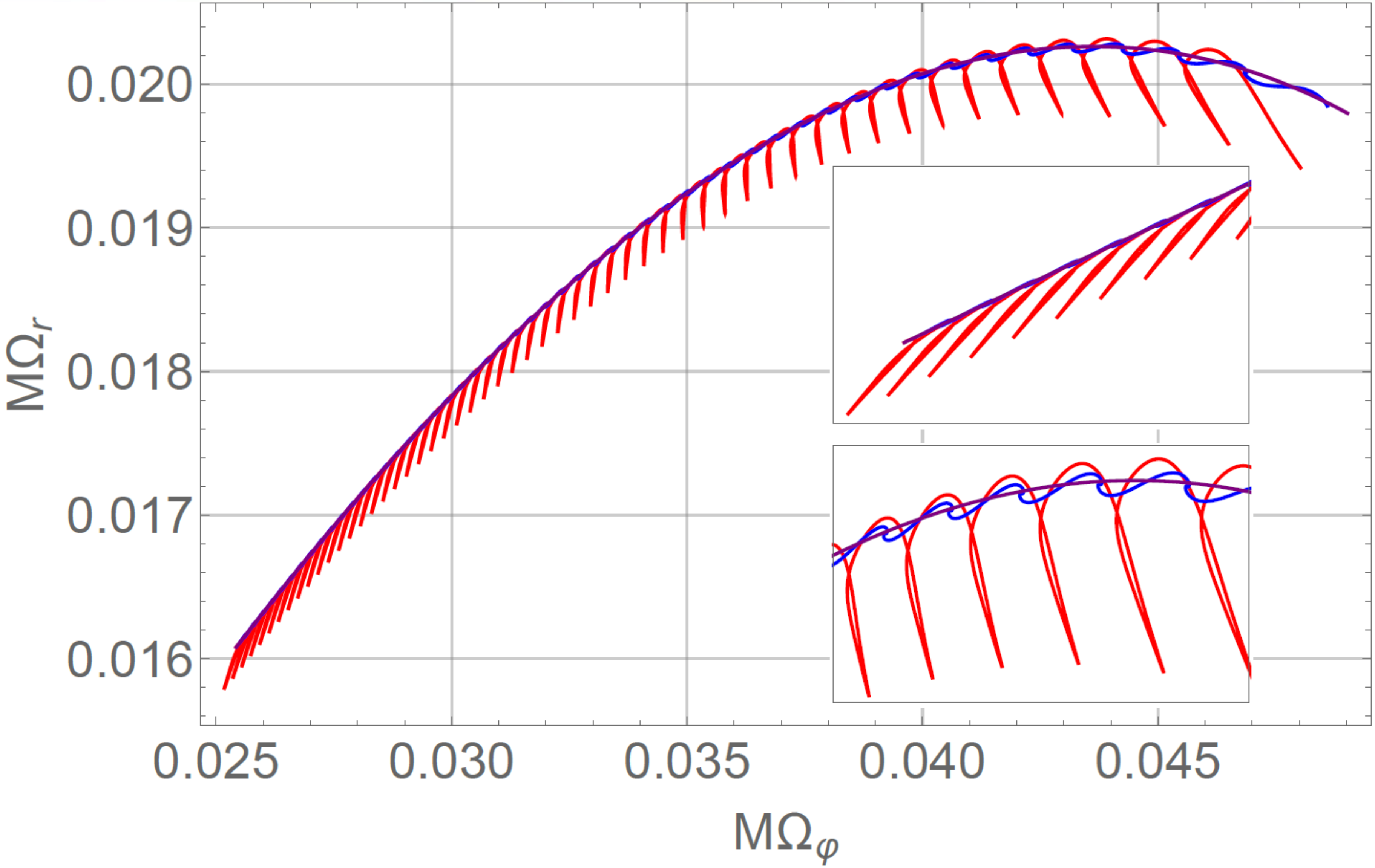
NORTH CAROLINA
space grant

UNC
THE GRADUATE SCHOOL

Dissertation Completion Fellowship

NSF

Evolution of gauge invariant freqs



Intermediate mass ratio inspiral