Radiation reaction effect on orbital parameters of a spinning particle in Kerr spacetime

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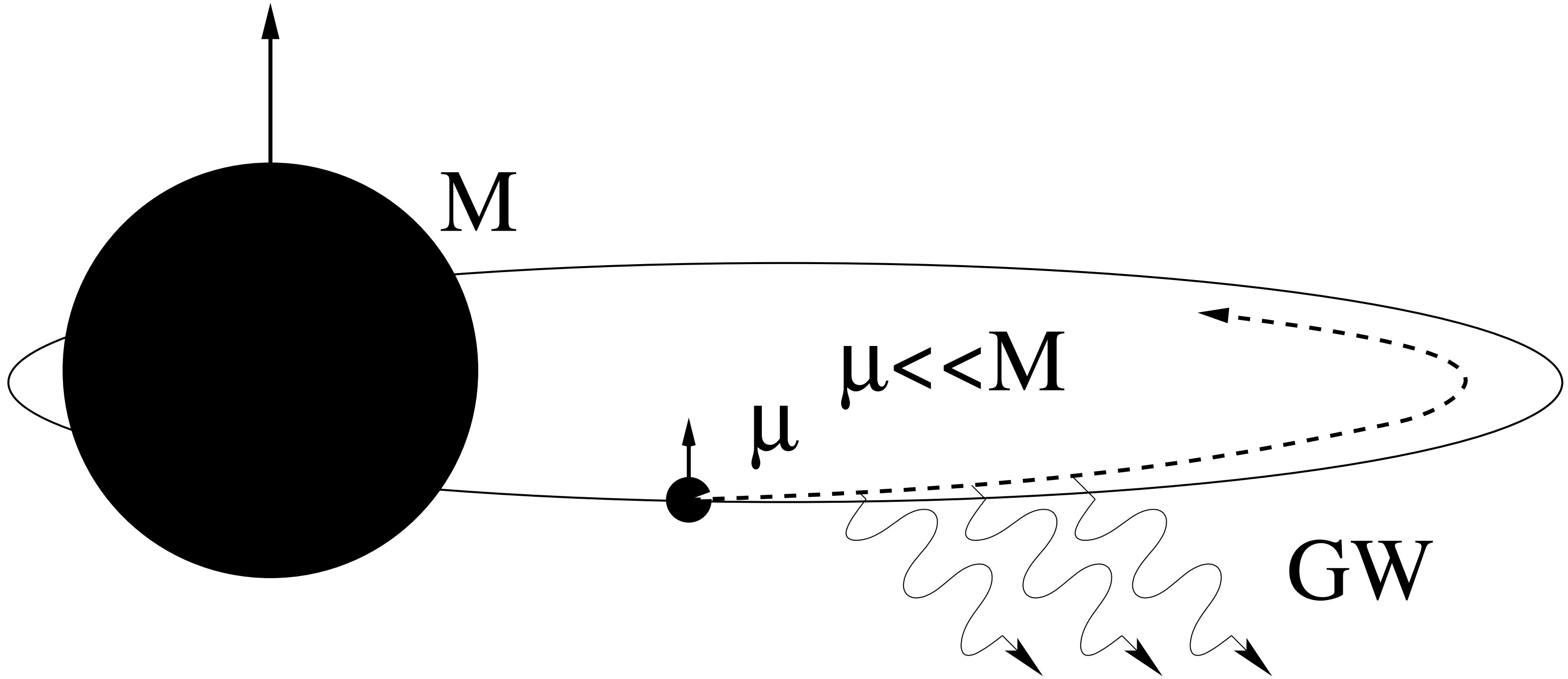
A spinning particle in Kerr spacetime



- Zeroth order in the mass ratio
  - Geodesic orbits with (E, L_z, C)
- First order in the mass ratio

  Deviation from the geodesic orbits because of
  - Radiation reaction
  - Spin of the particle, ...

Equations of motion of the spinning particle

- At the linear order in the spin:

$$\frac{D}{d\tau} p^\mu(\tau) = -\frac{1}{2} R^\mu{}_{\nu\rho\sigma}(z(\tau)) v^\nu(\tau) S^{\rho\sigma}(\tau),$$
$$\frac{D}{d\tau} S^{\mu\nu}(\tau) = 2p^{[\mu}(\tau) v^{\nu]}(\tau) (= 0),$$

where $v^\mu(\tau) = dz^\mu(\tau)/d\tau$, $p^\mu(\tau)$ and $S^{\mu\nu}(\tau)$ are the linear and spin angular momenta of the particle.

- ⋆ Hamiltonian can be found in Barausse+(2009), Vines+(2016).
- ⋆ Motion couples to the evolution of the spin
- ⋆ Frequencies of the motion and the spin precession?
- ⋆ Osculating geodesics? [Ruangsri+(2015)]

Conserved quantities

$$\frac{E}{\mu} = u^\mu \xi_\mu - \frac{1}{2\mu} S^{\mu\nu} \xi_{\mu;\nu}; \quad \frac{J_z}{\mu} = u^\mu \chi_\mu - \frac{1}{2\mu} S^{\mu\nu} \chi_{\mu;\nu},$$

$$\frac{Q}{\mu^2} = \frac{1}{2} f_{\mu\sigma} f_\nu{}^\sigma u^\mu u^\nu - u^\mu \frac{S^{\rho\sigma}}{\mu} \left(f^\nu{}_\sigma f_{\mu\rho\nu} - f_\mu{}^\nu f_{\rho\sigma\nu} \right),$$

where

$$\xi_\mu = \sqrt{\frac{\Delta}{\Sigma}}\, e^0{}_\mu + \frac{a \sin\theta}{\sqrt{\Sigma}}\, e^3{}_\mu; \quad \chi_\mu = a \sin^2\theta \sqrt{\frac{\Delta}{\Sigma}}\, e^0{}_\mu + \frac{(r^2 + a^2) \sin\theta}{\sqrt{\Sigma}}\, e^3{}_\mu,$$

$$f_{\mu\nu} = 2a \cos\theta\ e^1{}_{[\mu} e^0{}_{\nu]} + 2r\ e^2{}_{[\mu} e^3{}_{\nu]}; \quad f_{\mu\nu\sigma} = f_{\mu\nu;\sigma},$$

and $e^a{}_\mu$ is the tetrad frame deined by

$$e^0{}_\mu = \left(\sqrt{\frac{\Delta}{\Sigma}},\ 0,\ 0, -a \sin^2\theta \sqrt{\frac{\Delta}{\Sigma}} \right),\ e^1{}_\mu = \left(0,\ \sqrt{\frac{\Sigma}{\Delta}},\ 0,\ 0 \right),$$

$$e^2{}_\mu = \left(0,\ 0,\ \sqrt{\Sigma},\ 0 \right),\ e^3{}_\mu = \left(-\frac{a}{\sqrt{\Sigma}} \sin\theta,\ 0,\ 0,\ \frac{r^2 + a^2}{\sqrt{\Sigma}} \sin\theta \right),$$

Note $df_{\mu\nu\sigma}/dt = O(S^2)$ and Q is the approximate conserved quantity.
(cf. $K_{\mu\nu} = f_{\mu\sigma} f_\nu{}^\sigma = r^2 g_{\mu\nu} + 2\Sigma l_{(\mu} n_{\sigma)}$)

Equations of motion in the equatorial plane

Using conserved quantities E and J_z, EOM can be derived for spin-aligned binary in the equatorial plane

$$
\begin{aligned}
r^2 \frac{dt}{d\tau} &= a[J_z - (a+s)E] + \frac{r^2+a^2}{\Delta} P(E, J_z, a, s; r), \\
r^4 \left(\frac{dr}{d\tau}\right)^2 &= R(E, J_z, a, s; r), \\
r^2 \frac{d\phi}{d\tau} &= [J_z - (a+s)E] + \frac{a}{\Delta} P(E, J_z, a, s; r),
\end{aligned}
$$

where a is the spin of the Kerr and $s^2 = S_{\mu\nu}S^{\mu\nu}/2\mu^2$.
We can estimate orbital frequencies, Ω_r and Ω_ϕ, and compute energy flux using the energy-momentum tensor

$$
T^{ab} = \int d\tau \left\{ p^{(a} v^{b)} \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} - e^{(a}_{\ \nu} e^{b)}_{\ \rho} \nabla_\mu S^{\mu\nu} v^\rho \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} \right\},
$$

PN energy flux for spin-aligned binary in circular inspirals

	(spin)^1	(spin)^2	(spin)^3
BHP	2.5PN for μ's spin [Tanaka et.al.(1996)]	-	-
PN	3.5PN (NNLO) [Bohé et.al.(2013)]	3PN (NLO) [Bohé et.al.(2015)]	3.5PN (LO) [Marsat (2015)]

2.5PN energy flux for spin-alighed binary

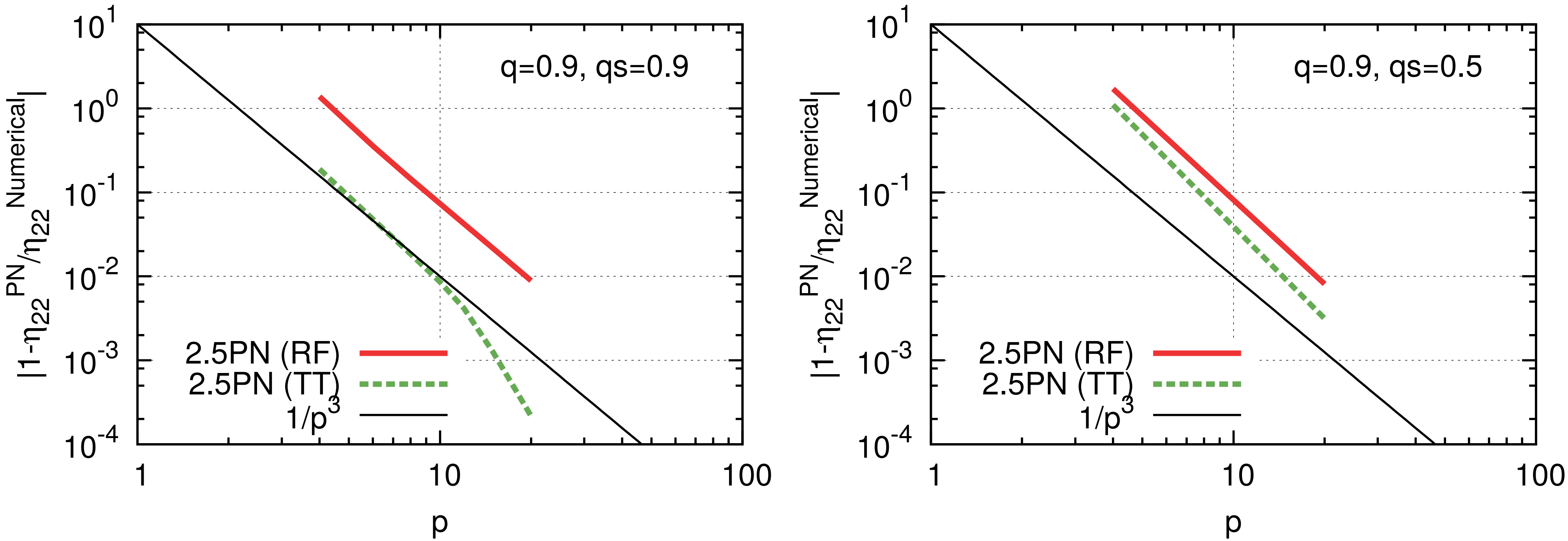
$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32}{5}\left(\frac{\mu}{M}\right)^2 (M\Omega)^{10/3} \times \left[1 - \frac{1247}{336}(M\Omega)^{2/3} + \left(4\pi - \frac{11}{4}q - \frac{5}{4}\hat{s}\right)(M\Omega) + \left(-\frac{44711}{9072} + \frac{33}{16}q^2 + \frac{31}{8}q\hat{s}\right)(M\Omega)^{4/3} + \left(-\frac{8191}{672}\pi + \frac{59}{16}q \underbrace{-\frac{13}{16}\hat{s}}_{\text{TBC...}}\right)(M\Omega)^{5/3} \right],$$

where $q = a/M$ and $\hat{s} = S/M$. [Tanaka et.al.(1996)]

- $q\,J_z$ terms may be larger than $\hat{s}\,J_z$ terms
- $q\hat{s}$ terms may be larger than q^2 terms

Comparison with numerical results

⋆ $|1 - dEdt^{\rm PN}/dEdt^{\rm Num}|$ for $q = 0.9, \hat{s} = 0.9,$ and $l = m = 2$

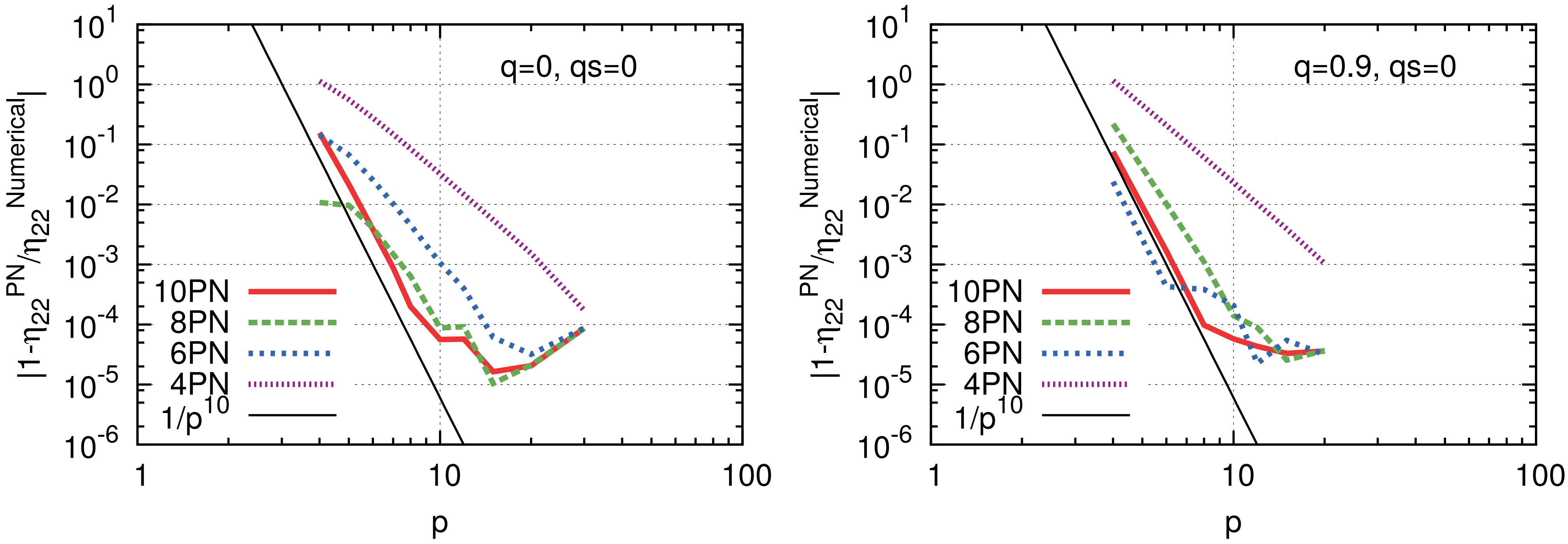


[Numerical results : Harms et. al. (2015)]

⇒ Relative error for Tanaka et. al. at 2.5PN is $\sim 1/p^3$ and better than RF

Comparison with numerical results

⋆ $|1 - dEdt^{\mathrm{PN}}/dEdt^{\mathrm{Num}}|$ for $q = 0.9, \hat{s} = 0,$ and $l = m = 2$



[PN results: RF (2015), Numerical results : Harms et. al. (2015)]

⇒ Relative error at 10PN is $\sim 1/p^{10}$ when $\hat{s} = 0$

Summary

- Spinning particle around a Kerr black hole
  - ⋆ Energy flux for circular and spin-aligned orbits
    - – Comparison with PN and numerical results suggest our results are not correct
- Future
  - ⋆ BH absorption
  - ⋆ High PN calculation for circular orbits in the equatorial plane
  - ⋆ Eccentric and spin-aligned orbits in the equatorial plane
  - ⋆ Circular and slightly inclined orbits
  - ⋆ More generic orbits?