Effective One-Body Theory and Self-Force

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In memoriam Steve Detweiler

19th Capra Meeting on Radiation Reaction in General Relativity
Observatoire de Paris, Meudon Campus, June 27-July 1, 2016

EOB THEORY + EOB[NR] + EOB[SF] DEVELOPMENTS

Buonanno,Damour 99	(2 PN Hamiltonian)
Buonanno,Damour 00	(Rad.Reac. full waveform)
Damour, Jaranowski,Schäfer 00	(3 PN Hamiltonian)
Damour 01, Buonanno, Chen, Damour 05, Damour-Jaranowski,Schäfer 08, Barausse, Buonanno, 10, Nagar 11, Balmelli-Jetzer 12, Taracchini et al 12,14, Damour,Nagar 14	(spin)
Damour, Nagar 07, Damour, Iyer, Nagar 08, Pan et al. 11	(factorized waveform)
Damour, Nagar 10 Bini-Damour-Faye 12	(tidal effects)
Bini, Damour 13, Damour, Jaranowski, Schäfer 15	(4 PN Hamiltonian)

EOB vs NR and EOB[NR]
Buonanno, Cook, Pretorius 07,
Buonanno, Pan, Taracchini 08-
Damour-Nagar 08-

EOB vs SF and EOB[SF]
Damour 09
Barack-Sago-Damour 10
Barausse-Buonanno-LeTiec 12
Akcay-Barack-Damour-Sago 12
Bini-Damour 13-16
LeTiec 15
Bini-Damour-Geralico 16
Hopper-Kavanagh-Ottewill 16
Akcay-vandeMeent 16

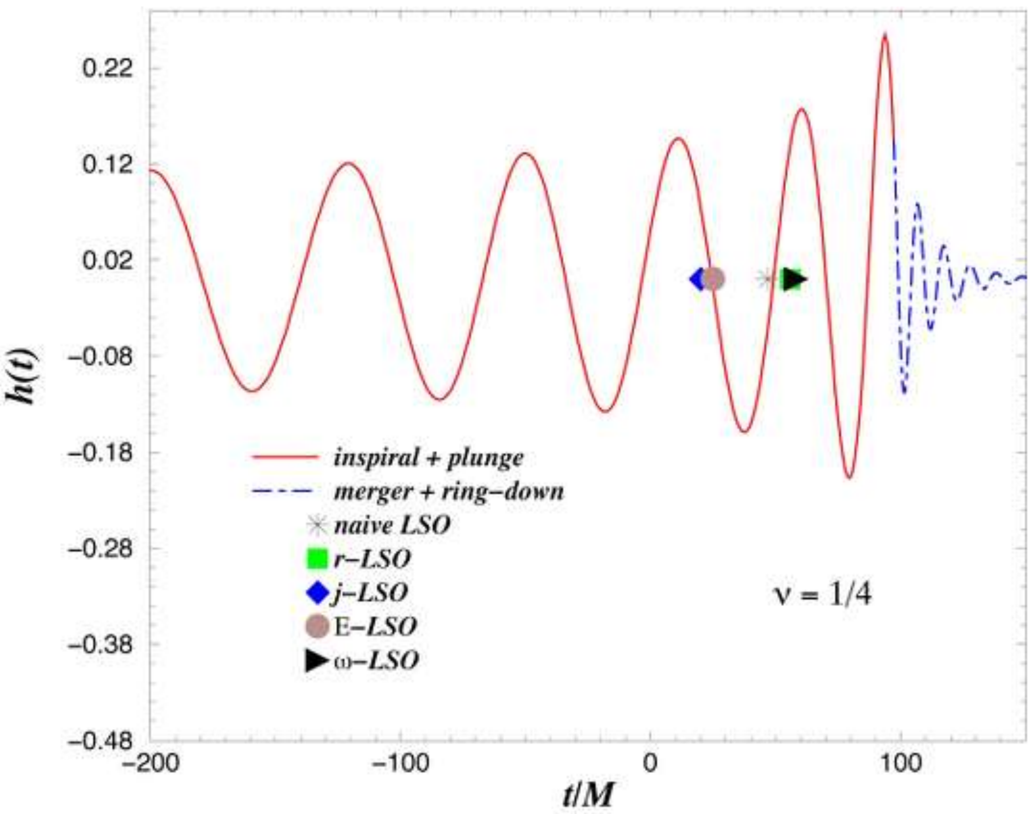
Reduced Order Model version (Pürrer 2014, 2016) of EOB[NR] (Taracchini et al 2014)

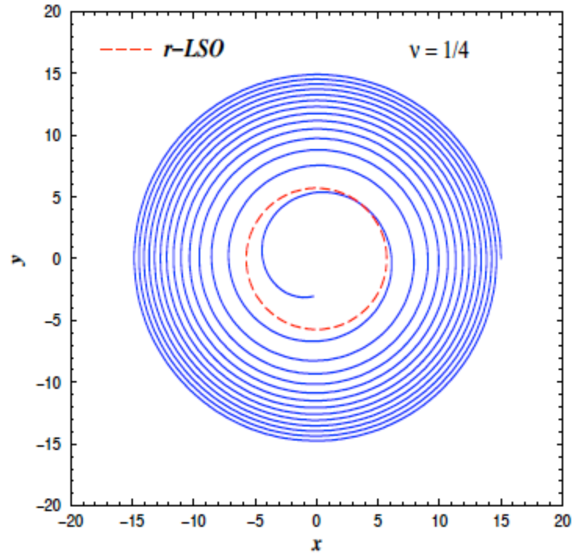
Phenomenological model (Ajith et al 2007, Hannam et al 2014, Husa et al 2016, Kahn et al 2016) of FFT of hybrids EOB + NR

EFFECTIVE ONE BODY (EOB) FORMALISM

Buonanno-Damour 99,00; Damour-Jaranowski-Schaefer 00; Damour 01; Buonanno-Chen-Damour 05

Buonanno-Damour 2000

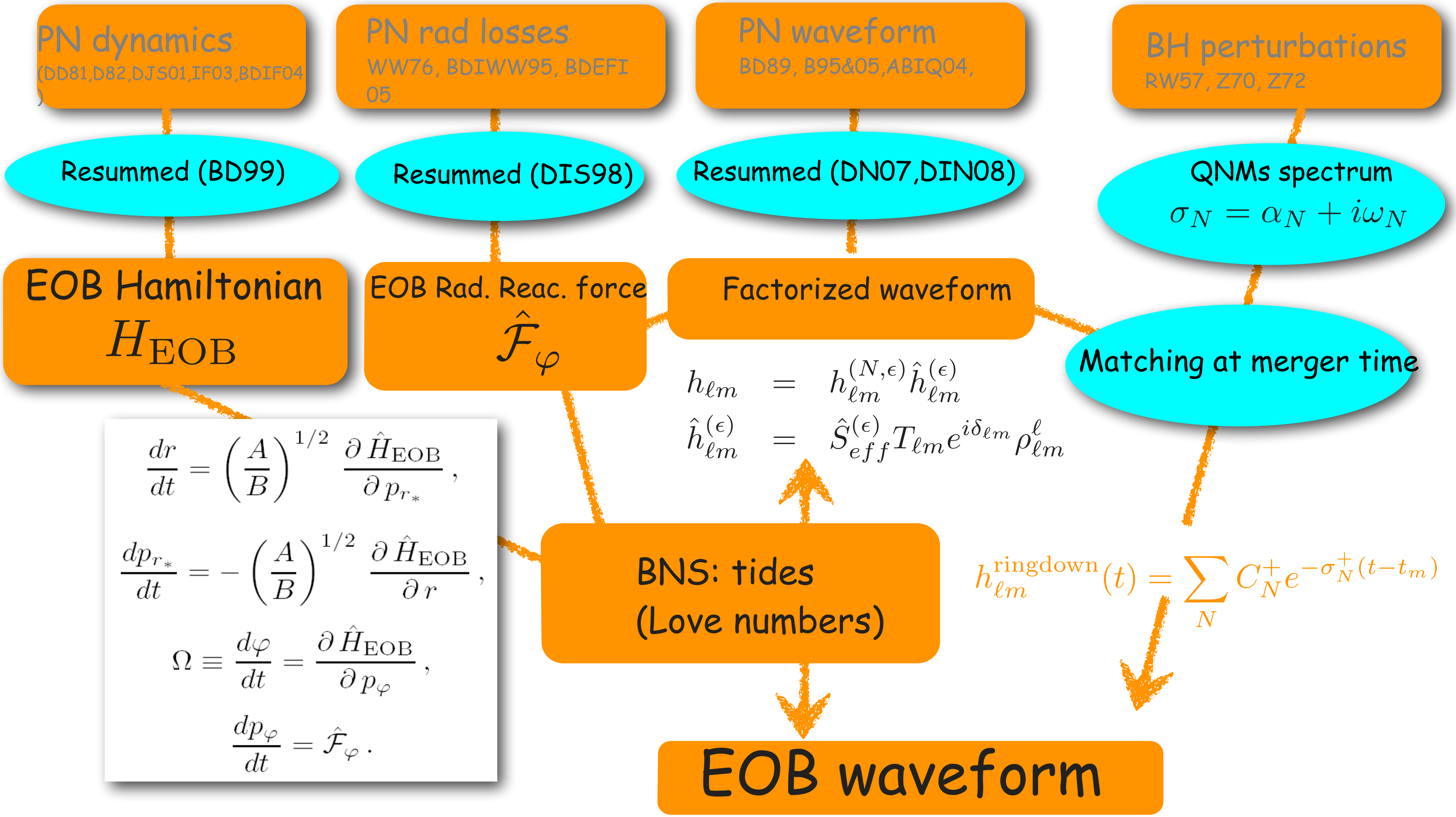




- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2} = \frac{\mu}{M}$$

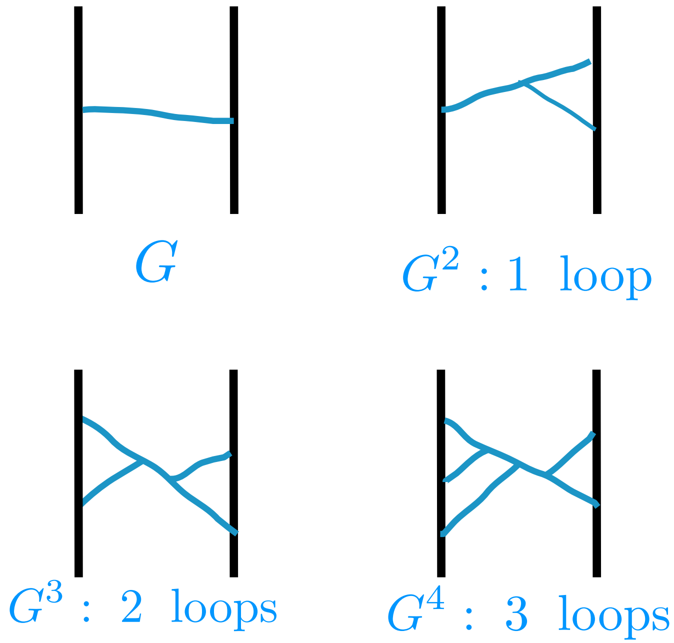
STRUCTURE OF THE EOB FORMALISM



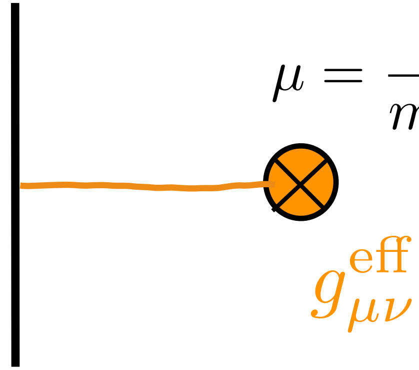
$$h^{\rm EOB}_{\ell m} = \theta(t_m - t)\, h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m)\, h^{\rm ringdown}_{\ell m}(t)$$

REAL DYNAMICS VERSUS EFFECTIVE DYNAMICS

Real dynamics



Effective dynamics



$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$S = -\int \mu ds + \ldots$$

$$H = H_0 + \left(G H_1 + \frac{G^2}{c^2} H_2 + \frac{G^3}{c^4} H_3 + \frac{G^4}{c^6} H_4 \right) \left(1 + \frac{1}{c^2} + \ldots \right)$$

Effective point-particle moving in a (spherical) effective metric

$$ds^2_{\mathrm{eff}} = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

TWO-BODY/EOB "CORRESPONDENCE":

BREZIN-ITZYKSON-ZINN-JUSTIN + THINK QUANTUM-MECHANICALLY (J.A. WHEELER)

Real 2-body system
(in the c.o.m. frame)
(m_1, m_2)

1:1 map

An effective particle $\mu = \frac{m_1 m_2}{m_1 + m_2}$
in some effective metric $g_{\mu\nu}^{\mathrm{eff}}$

$\mathcal{E} = f(E)$

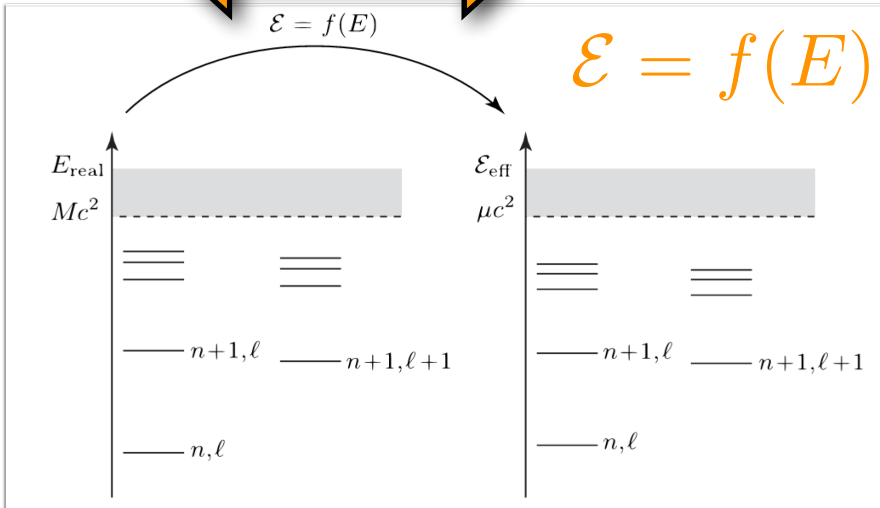


Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

$$\mu^2 + g_{\mathrm{eff}}^{\mu\nu} \frac{\partial S_{\mathrm{eff}}}{\partial x^\mu} \frac{\partial S_{\mathrm{eff}}}{\partial x^\nu} + \mathcal{O}(p^4) = 0$$

Sommerfeld's
"Old Quantum Mechanics"
(action-angle variables &
Delaunay Hamiltonian)

$$J = \ell\hbar = \frac{1}{2\pi} \oint p_\varphi d\varphi$$

$$N = n\hbar = I_r + J$$

$$I_r = \frac{1}{2\pi} \oint p_r dr$$

$$H^{\mathrm{classical}}(q,p) \longrightarrow H^{\mathrm{classical}}(I_a) \longrightarrow \boxed{E^{\mathrm{quantum}}(I_a = n_a h) = f^{-1}[\mathcal{E}_{\mathrm{eff}}^{\mathrm{quantum}}(I_a^{\mathrm{eff}} = n_a h)]}$$

EXPLICIT FORM OF THE 3PN EOB HAMILTONIAN

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$$

$$\nu \equiv \frac{\mu}{M} \equiv \frac{m_1 m_2}{(m_1+m_2)^2}$$

All functions are a ν-dependent deformation of the Schwarzschild ones

$$A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

Crucial EOB radial potential

Contribution at 3PN

2-body Taylor-expanded N + 1PN + 2PN Hamiltonian

$$H_{\mathrm{N}}(\mathbf{x}_a, \mathbf{p}_a) = \frac{\mathbf{p}_1^2}{2m_1} - \frac{1}{2}\frac{Gm_1m_2}{r_{12}} + (1 \leftrightarrow 2)$$

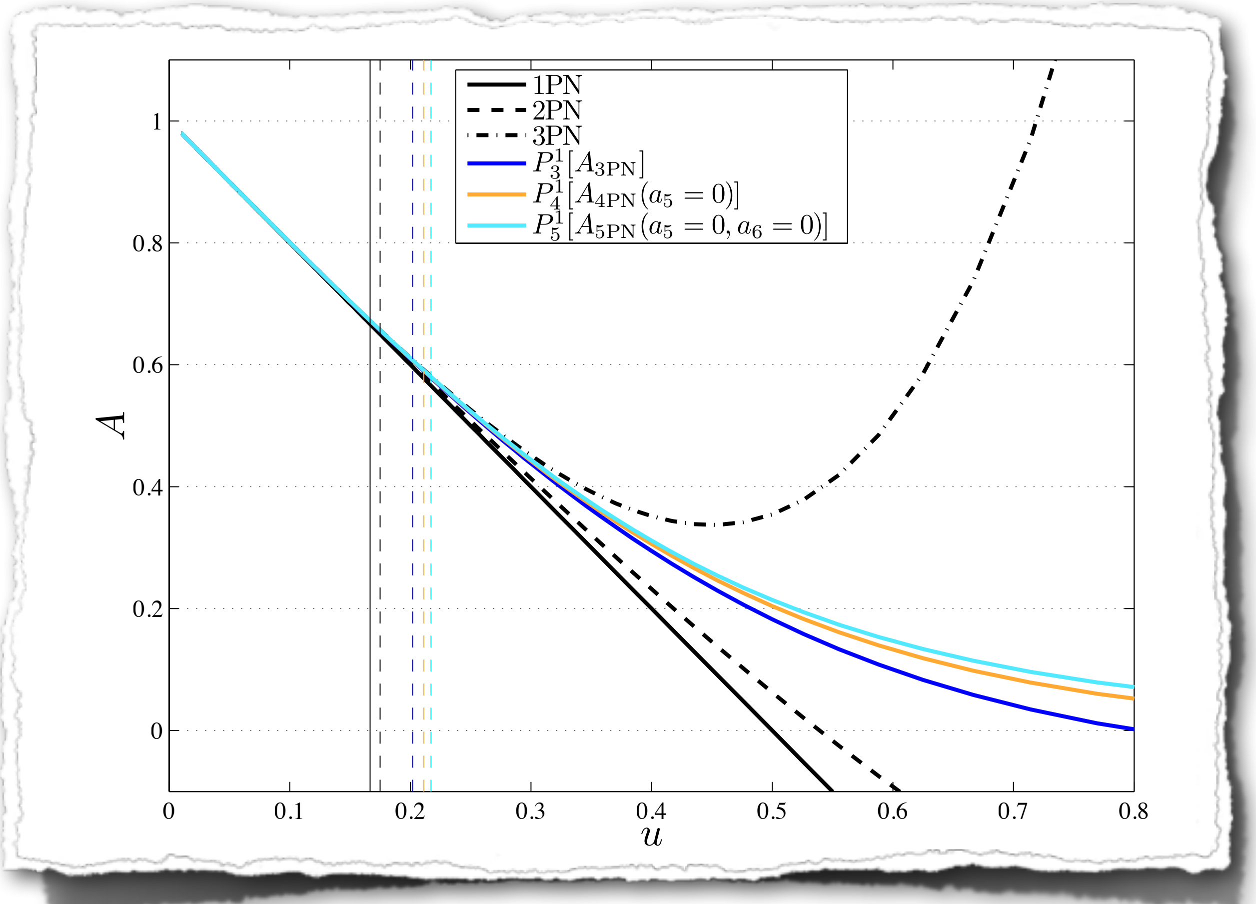
$$c^2 H_{1\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = -\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(-12\frac{\mathbf{p}_1^2}{m_1^2} + 14\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2} + 2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right) + \frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}} + (1 \leftrightarrow 2),$$

$$\begin{aligned} c^4 H_{2\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = {} & \frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(5\frac{(\mathbf{p}_1^2)^2}{m_1^4} - \frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2} - \frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2} + 5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right. \\ & \left. - 6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2} - \frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right) \\ & + \frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left(m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2} + 19\frac{\mathbf{p}_2^2}{m_2^2}\right) - \frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2) + 6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right) \\ & - \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2 + 5m_1m_2 + m_2^2)}{r_{12}^2} + (1 \leftrightarrow 2), \end{aligned}$$

2-body Taylor-expanded 3PN Hamiltonian [JS 98, DJS 01]

$$
\begin{aligned}
c^6 H_{3\mathrm{PN}}(\mathbf{x}_a,\mathbf{p}_a) = &-\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7} + \frac{1}{32}\frac{Gm_1m_2}{r_{12}}\left(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6} + 4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2+4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4m_2^2} + 6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4m_2^2}\right.\\
&-10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2+\mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4m_2^2} + 24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4m_2^2}\\
&+2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3m_2^3} + \frac{(7\mathbf{p}_1^2\mathbf{p}_2^2-10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3m_2^3}\\
&+\frac{(\mathbf{p}_1^2\mathbf{p}_2^2-2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3m_2^3} + 15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3m_2^3}\\
&\left.-18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3m_2^3} + 5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3m_2^3}\right) + \frac{G^2m_1m_2}{r_{12}^2}\left(\frac{1}{16}(m_1-27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4}\right.\\
&-\frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3m_2} + \frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2+371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2} + \frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3} + \frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3}\\
&-\frac{1}{8}m_1\frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)+11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3m_2} - \frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3m_2}\\
&+\frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2} + \frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\\
&\left.-\frac{1}{48}(220m_1+193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right) + \frac{G^3m_1m_2}{r_{12}^3}\left(-\frac{1}{48}\left(425m_1^2+\left(473-\frac{3}{4}\pi^2\right)m_1m_2+150m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2}\right.\\
&+\frac{1}{16}\left(77(m_1^2+m_2^2)+\left(143-\frac{1}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2} + \frac{1}{16}\left(20m_1^2-\left(43+\frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2}\\
&\left.+\frac{1}{16}\left(21(m_1^2+m_2^2)+\left(119+\frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right)\\
&+\frac{1}{8}\frac{G^4m_1m_2^3}{r_{12}^4}\left(\left(\frac{227}{3}-\frac{21}{4}\pi^2\right)m_1+m_2\right) + (1\leftrightarrow 2).
\end{aligned}
$$

PADE RESUMMATION OF A(R) POTENTIAL



- Continuity with Schwarzschild metric: A(r) needs to have a zero
- Simple (possible) prescription: use a Padé representation of the potential

$$A(r) = P^1_3[A^{3PN}(r)] = \frac{1+n_1 u}{1+d_1 u + d_2 u^2 + d_3 u^3}$$

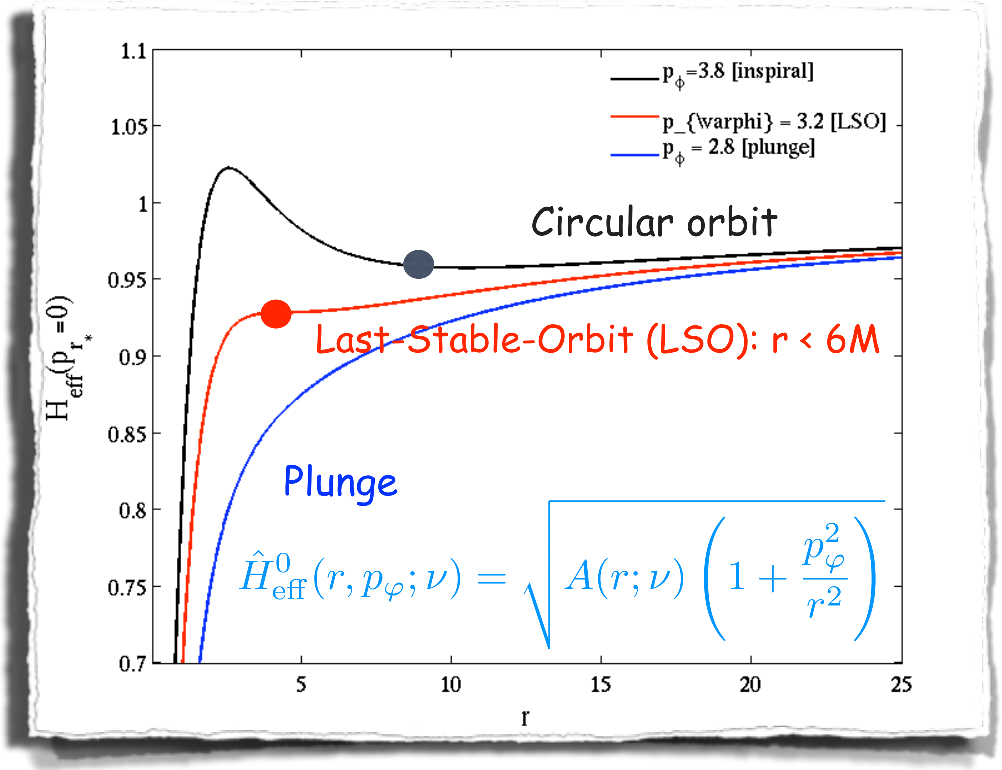
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{\varphi}} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_{\varphi} = \hat{\mathcal{F}}_{\varphi}$$



- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_{\varphi}^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Plus horizon contribution [Nagar&Akcay2012]

Resummation multipole by multipole
(Damour&Nagar 2007,
Damour, Iyer & Nagar 2008,
Damour & Nagar, 2009)

MULTIPOLAR WAVEFORM RESUMMATION

Resummation of the waveform (and flux) multipole by multipole

[Damour&Nagar 2007, Damour, Iyer, Nagar 2008]

Next-to-quasi-circular correction

$$h_{\ell m} \equiv h_{\ell m}^{(N,\epsilon)} \hat{h}_{\ell m}^{(\epsilon)} \hat{h}_{\ell m}^{\mathrm{NQC}}$$

Newtonian × PN × NQC

PN-correction

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{\mathrm{eff}}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

Remnant phase and modulus corrections: "improved" PN series

The "Tail factor"

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi \hat{k}} e^{2i\hat{\hat{k}} \ln(2kr_0)}$$

Resums an infinite number of leading logarithms in tail effects (hereditary contributions)

Effective source:

EOB (effective) energy (even-parity modes)

EOB angular momentum (odd-parity modes)

RESUMMING RADIATION REACTION

$$\mathcal{F}_\varphi \equiv -\frac{1}{8\pi\Omega}\sum_{\ell=2}^{\ell_{\rm max}}\sum_{m=1}^{\ell}(m\Omega)^2|Rh_{\ell m}^{(\epsilon)}|^2$$

Residual amplitude correction:

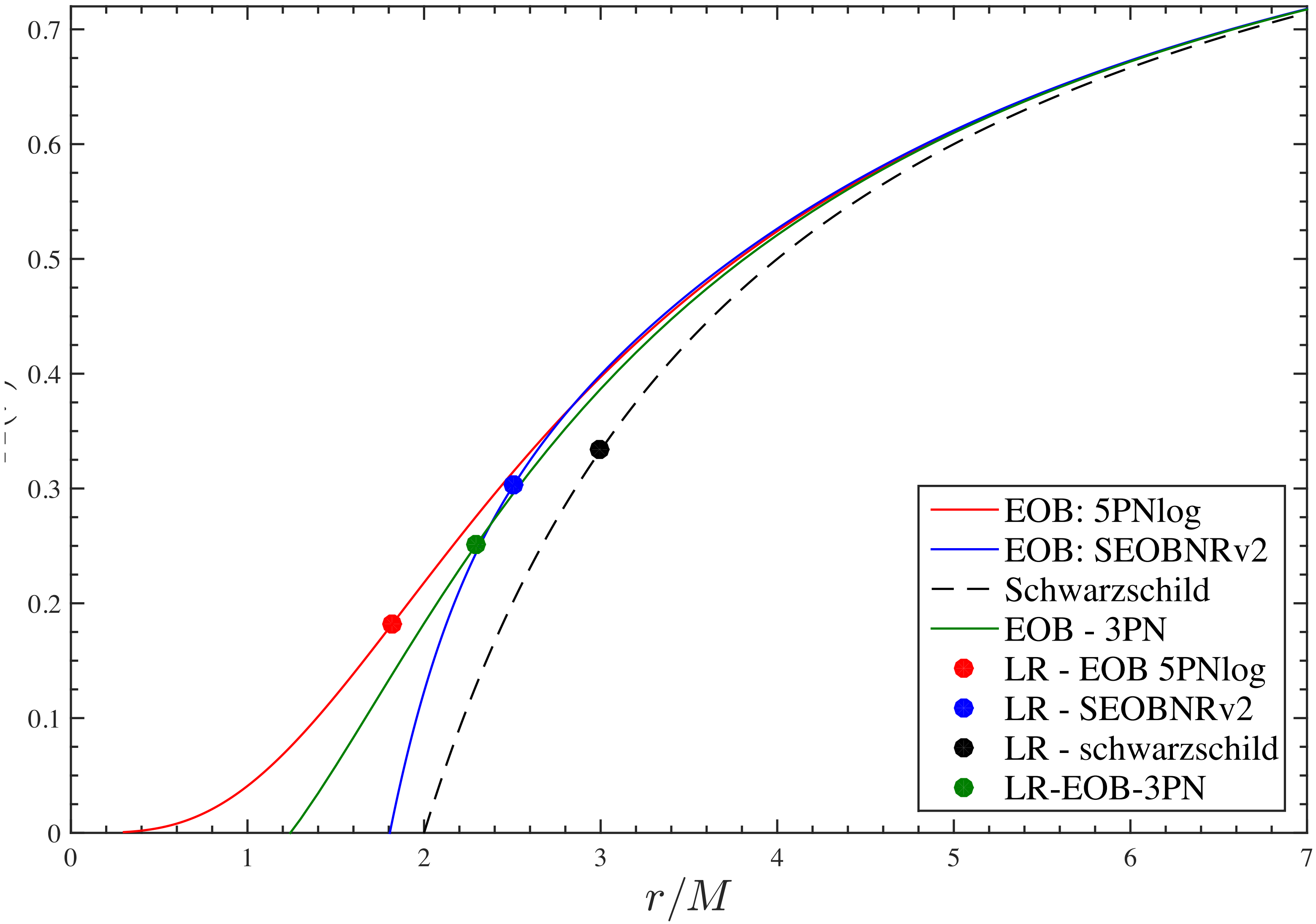
$$\begin{aligned}\rho_{22}(x;\nu) = 1 &+ \left(\frac{55\nu}{84}-\frac{43}{42}\right)x + \left(\frac{19583\nu^2}{42336}-\frac{33025\nu}{21168}-\frac{20555}{10584}\right)x^2\\ &+ \left(\frac{10620745\nu^3}{39118464}-\frac{6292061\nu^2}{3259872}+\frac{41\pi^2\nu}{192}-\frac{48993925\nu}{9779616}-\frac{428}{105}\mathrm{eulerlog}_2(x)+\frac{1556919113}{122245200}\right)x^3\\ &+ \left(\frac{9202}{2205}\mathrm{eulerlog}_2(x)-\frac{387216563023}{160190110080}\right)x^4+\left(\frac{439877}{55566}\mathrm{eulerlog}_2(x)-\frac{16094530514677}{533967033600}\right)x^5+\mathcal{O}(x^6),\end{aligned}$$

$$\mathrm{eulerlog}_m(x) = \gamma + \log 2 + \frac{1}{2}\log x + \log m$$

MAIN RADIAL RADIAL EOB POTENTIAL A(R)

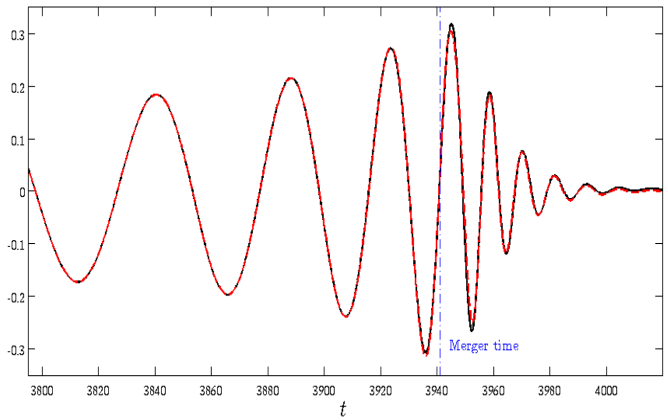
m1=m2 case

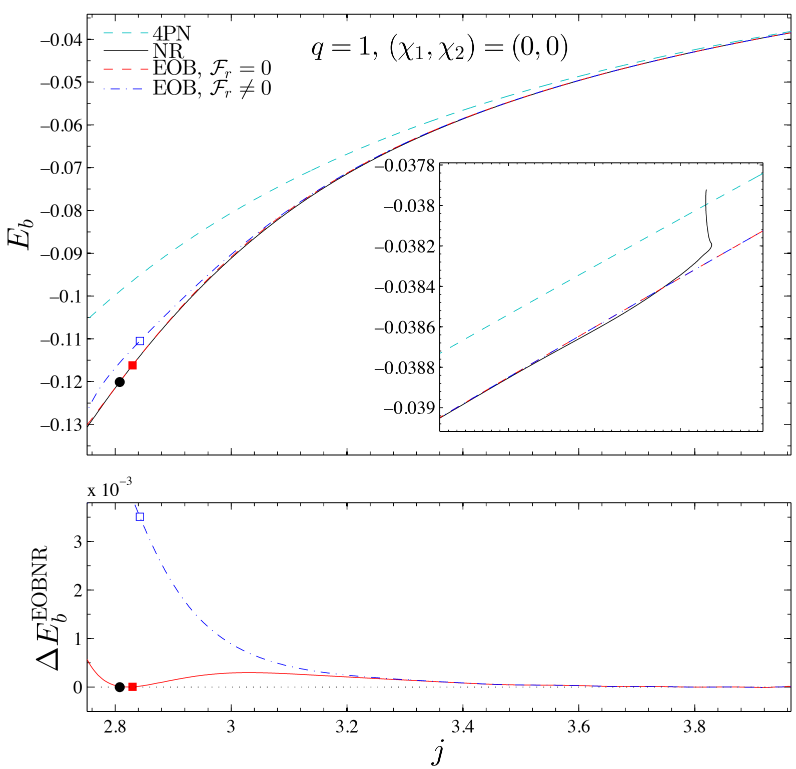
A(r)

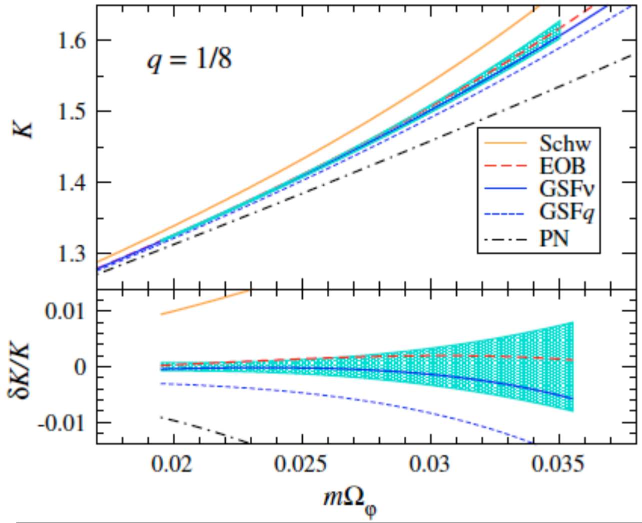


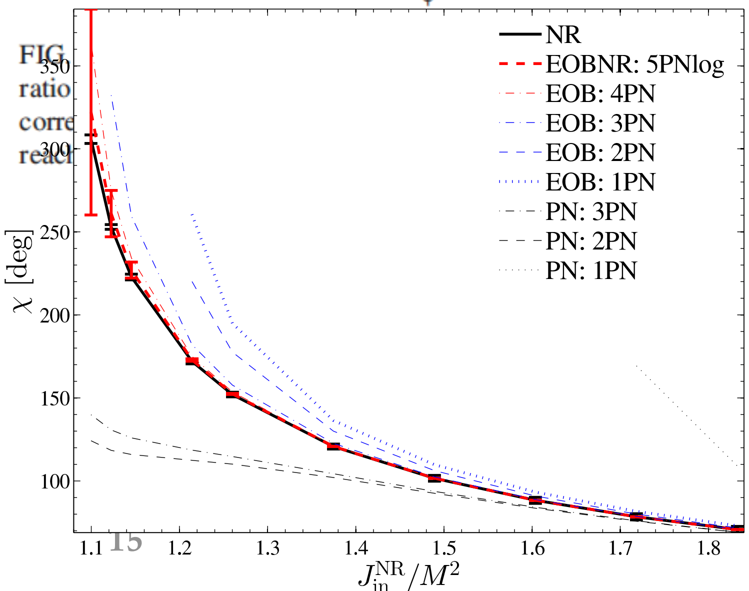
EOB VS NR

waveform (Damour-Nagar 09),
energetics(Nagar-Damour-Resswig-Pollney 16),
periastron precession (LeTiec-Mroue-Barack-Buonanno-Pfeiffer-Sago-Tarachini 11, and
scattering angle (Damour-Guercilena-Hinder-Hopper-Nagar-Rezzolla 14)







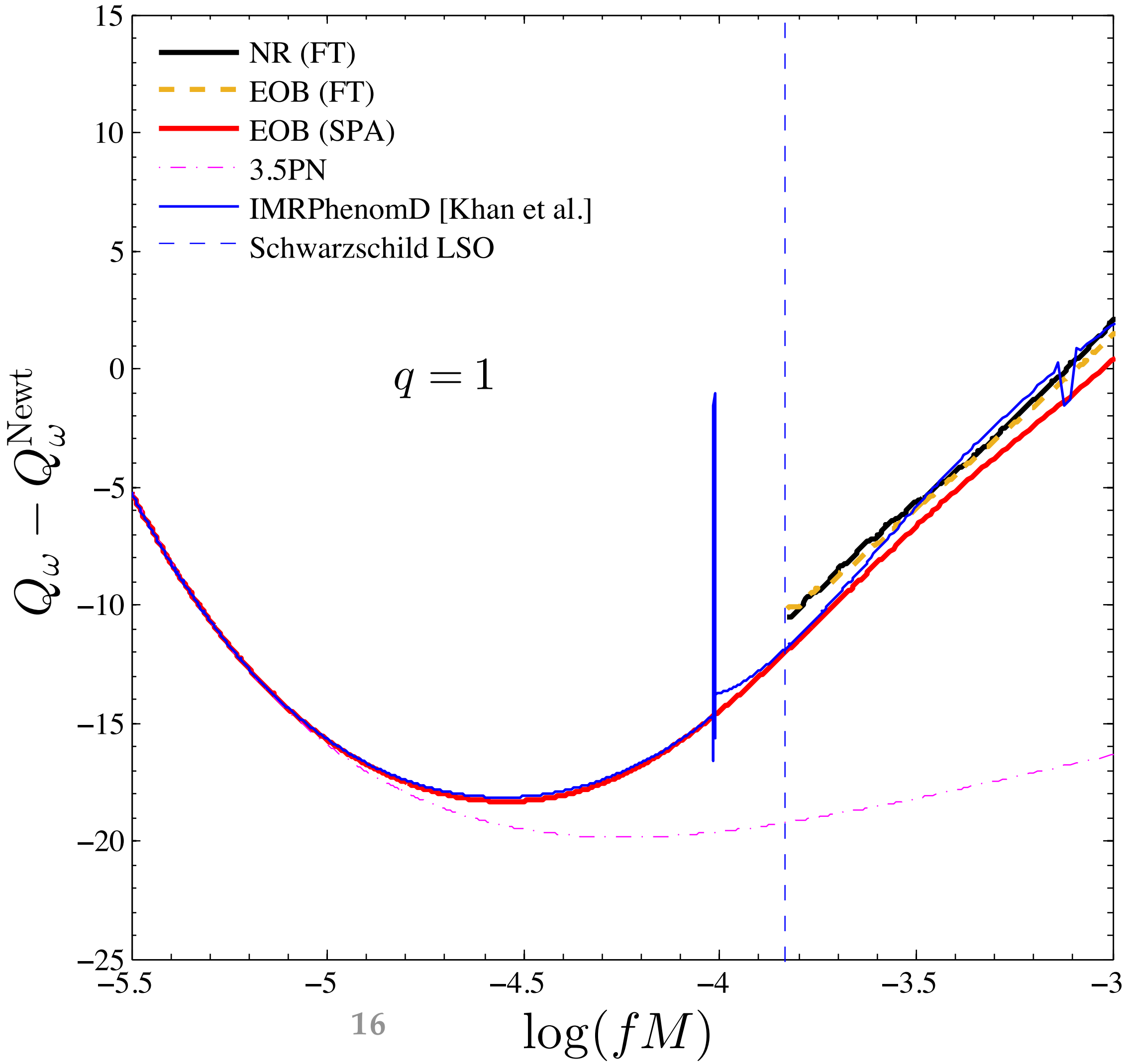


PN, EOB, NR, PHENOMD

PN accuracy loss during inspiral

Dimensionless « quality factor » of GW phase $$Q_\omega = f^2 \frac{d^2\psi(f)}{df^2} \approx \frac{\omega^2}{\dot{\omega}}$$

$$Q_\omega - Q_\omega^N$$



THE EOB POTENTIALS: A, B, Q,G_S, G_S*

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$H = M\sqrt{1 + 2\nu\left(\frac{H_{\text{eff}}}{\mu} - 1\right)}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$\mathbf{S}_* = \frac{m_2}{m_1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2$$

Aligned spins:

$$H_{\text{eff}} = \sqrt{A(r, \nu, S, S_*)\left(\mu^2 + \frac{L^2}{r^2} + \frac{p_r^2}{B(r)} + Q(r, p_r)\right)} + G_S \mathbf{L}.\mathbf{S} + G_{S_*}\mathbf{L}.\mathbf{S}_*$$

EOB, PN AND SF

PN vs SF expansions :expansion in $u = \dfrac{GM}{c^2 r}$

vs expansion in $\nu = \dfrac{\mu}{M} = \dfrac{m_1 m_2}{(m_1 + m_2)^2}$

$$A^{\mathrm{PN}}(u;\nu) = 1 - 2u + A_2(\nu)u^2 + A_3(\nu)u^3 + A_4(\nu)u^4 + O_{\ln}(u^5)$$

$$A^{\mathrm{SF}}(u;\nu) = 1 - 2u + \nu a_1(u) + \nu^2 a_2(u) + O(\nu^3)$$

$$\bar{D}(u) = (A(u)B(u))^{-1}$$

$$\bar{D}^{\mathrm{SF}}(u) = 1 + \nu \bar{d}_1(u) + \nu^2 \bar{d}_2(u) + 0(\nu^3)$$

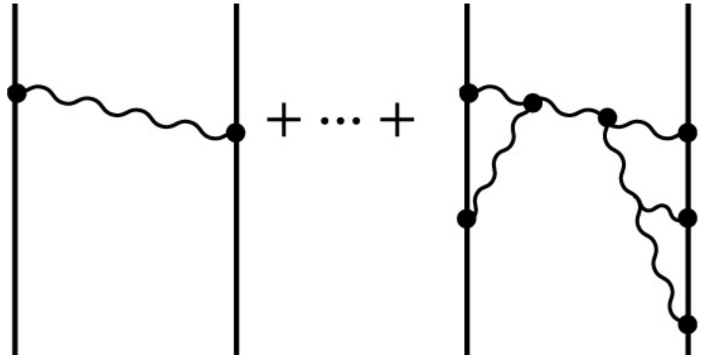
$$\frac{Q(u, p_r, \nu)}{\mu^2} = \nu q_4^{1SF}(u) \left(\frac{p_r}{\mu}\right)^4 + ...$$

$$M^3 G_S(u;\nu) = 2u^3 + \nu G_S^{1SF}(u) + ...$$

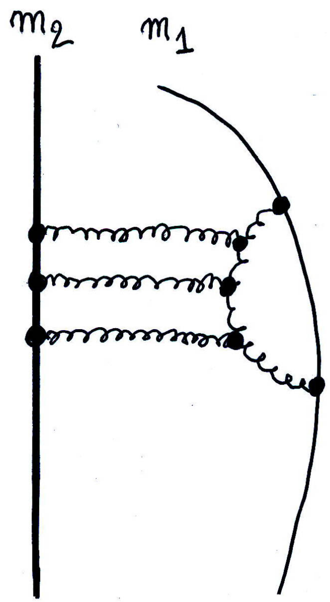
$$M^3 G_{S_*}(u;\nu) = G_{S_*}^{\text{test spinning particle}}(u) + \nu G_{S_*}^{1SF}(u) + ...$$

PN, EOB AND GSF

Comparable-mass case: $m_1 \sim m_2$



Gravitational Self-Force Theory : $m_1 << m_2$



- Analytical high-PN results : Blanchet-Detweiler-LeTiec-Whiting '10, Damour '10, Blanchet et al '10, LeTiec et al '12, Bini-Damour '13-15, Kavanagh-Ottewill-Wardell '15
- (gauge-invariant) Numerical results : Detweiler '08, Barack-Sago '09, Blanchet-Detweiler-LeTiec-Whiting '10, Barack-Damour-Sago '10, Shah-Friedman-Keidl '12, Dolan et al '14, Nolan et al '15, …
- Analytical PN results from high-precision (hundreds to thousands of digits !) numerical results : Shah-Friedman-Whiting '14, Johnson-McDaniel-Shah-Whiting '15

4PN ADM HAMILTONIAN (DJS 14)

(with a crucial SF input from Bini-Damour 13)

$$
\begin{aligned}
\widehat{H}_{4\mathrm{PN}}(\mathbf{r},\mathbf{p}) &= \left(\frac{7}{256}-\frac{63}{256}\nu+\frac{189}{256}\nu^2-\frac{105}{128}\nu^3+\frac{63}{256}\nu^4\right)(\mathbf{p}^2)^5 \\
&+ \left\{\frac{45}{128}(\mathbf{p}^2)^4-\frac{45}{16}(\mathbf{p}^2)^4\,\nu+\left(\frac{423}{64}(\mathbf{p}^2)^4-\frac{3}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3-\frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2\right)\nu^2\right. \\
&+ \left(-\frac{1013}{256}(\mathbf{p}^2)^4+\frac{23}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3+\frac{69}{128}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2-\frac{5}{64}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2+\frac{35}{256}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^3 \\
&+ \left.\left(-\frac{35}{128}(\mathbf{p}^2)^4-\frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^3-\frac{9}{64}(\mathbf{n}\cdot\mathbf{p})^4(\mathbf{p}^2)^2-\frac{5}{32}(\mathbf{n}\cdot\mathbf{p})^6\mathbf{p}^2-\frac{35}{128}(\mathbf{n}\cdot\mathbf{p})^8\right)\nu^4\right\}\frac{1}{r} \\
&+ \left\{\frac{13}{8}(\mathbf{p}^2)^3+\left(-\frac{791}{64}(\mathbf{p}^2)^3+\frac{49}{16}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2-\frac{889}{192}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2+\frac{369}{160}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu\right. \\
&+ \left(\frac{4857}{256}(\mathbf{p}^2)^3-\frac{545}{64}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2+\frac{9475}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2-\frac{1151}{128}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^2 \\
&+ \left.\left(\frac{2335}{256}(\mathbf{p}^2)^3+\frac{1135}{256}(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2-\frac{1649}{768}(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2+\frac{10353}{1280}(\mathbf{n}\cdot\mathbf{p})^6\right)\nu^3\right\}\frac{1}{r^2} \\
&+ \left\{\frac{105}{32}(\mathbf{p}^2)^2+\left(\left(\frac{2749\pi^2}{8192}-\frac{589189}{19200}\right)(\mathbf{p}^2)^2+\left(\frac{63347}{1600}-\frac{1059\pi^2}{1024}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2+\left(\frac{375\pi^2}{8192}-\frac{23533}{1280}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu\right. \\
&+ \left(\left(\frac{18491\pi^2}{16384}-\frac{1189789}{28800}\right)(\mathbf{p}^2)^2+\left(-\frac{127}{3}-\frac{4035\pi^2}{2048}\right)(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2+\left(\frac{57563}{1920}-\frac{38655\pi^2}{16384}\right)(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^2 \\
&+ \left.\left(-\frac{553}{128}(\mathbf{p}^2)^2-\frac{225}{64}(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2-\frac{381}{128}(\mathbf{n}\cdot\mathbf{p})^4\right)\nu^3\right\}\frac{1}{r^3} \\
&+ \left\{\frac{105}{32}\mathbf{p}^2+\left(\left(\frac{185761}{19200}-\frac{21837\pi^2}{8192}\right)\mathbf{p}^2+\left(\frac{3401779}{57600}-\frac{28691\pi^2}{24576}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu\right. \\
&+ \left.\left(\left(\frac{672811}{19200}-\frac{158177\pi^2}{49152}\right)\mathbf{p}^2+\left(\frac{110099\pi^2}{49152}-\frac{21827}{3840}\right)(\mathbf{n}\cdot\mathbf{p})^2\right)\nu^2\right\}\frac{1}{r^4} \\
&+ \left\{-\frac{1}{16}+\left(\frac{6237\pi^2}{1024}-\frac{169199}{2400}\right)\nu+\left(\frac{7403\pi^2}{3072}-\frac{1256}{45}\right)\nu^2\right\}\frac{1}{r^5} \\
&- \frac{1}{5}\,\frac{G^2M}{c^8}\,I^{(3)}_{ij}(t)\times\mathrm{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{\mathrm{d}v}{|v|}\,I^{(3)}_{ij}(t+v)
\end{aligned}
$$

4PN EOB POTENTIALS (DJS 15)

Remarkable feature: after several cancellations A3PN(u,nu) is linear in nu, and A4PN(u,nu) is only quadratic in nu

$$A(u)=1-2u+2\nu u^3+\left(\frac{94}{3}-\frac{41\pi^2}{32}\right)\nu u^4+\left(\left(\frac{2275\pi^2}{512}-\frac{4237}{60}+\frac{128}{5}\gamma_{\rm E}+\frac{256}{5}\ln 2\right)\nu+\left(\frac{41\pi^2}{32}-\frac{221}{6}\right)\nu^2+\frac{64}{5}\nu\ln u\right)u^5, \tag{8.1a}$$

$$\bar{D}(u)=1+6\nu u^2+(52\nu-6\nu^2)u^3+\left(\left(-\frac{533}{45}-\frac{23761\pi^2}{1536}+\frac{1184}{15}\gamma_{\rm E}-\frac{6496}{15}\ln 2+\frac{2916}{5}\ln 3\right)\nu\right.$$
$$\left.+\left(\frac{123\pi^2}{16}-260\right)\nu^2+\frac{592}{15}\nu\ln u\right)u^4, \tag{8.1b}$$

$$\hat{Q}(\mathbf{r}',\mathbf{p}')=\left(2(4-3\nu)\nu u^2+\left(\left(-\frac{5308}{15}+\frac{496256}{45}\ln 2-\frac{33048}{5}\ln 3\right)\nu-83\nu^2+10\nu^3\right)u^3\right)(\mathbf{n}'\cdot\mathbf{p}')^4$$
$$+\left(\left(-\frac{827}{3}-\frac{2358912}{25}\ln 2+\frac{1399437}{50}\ln 3+\frac{390625}{18}\ln 5\right)\nu-\frac{27}{5}\nu^2+6\nu^3\right)u^2(\mathbf{n}'\cdot\mathbf{p}')^6+\mathcal{O}[\nu u(\mathbf{n}'\cdot\mathbf{p}')^8]. \tag{8.1c}$$

SF-DERIVED EOB INFORMATION

Damour 2010: use gauge-invariant observables

1SF contribution to LSO frequency shift: $M\Omega_{\rm LSO} = \frac{1}{6^{3/2}}(1 + C_\Omega \nu + O(\nu^2))$

$$C_\Omega = 1 - \sqrt{\frac{8}{9}} + \frac{3}{2}\left(a_1(\frac{1}{6}) + \frac{1}{6}a_1'(\frac{1}{6}) + \frac{1}{18}a_1''(\frac{1}{6})\right)$$

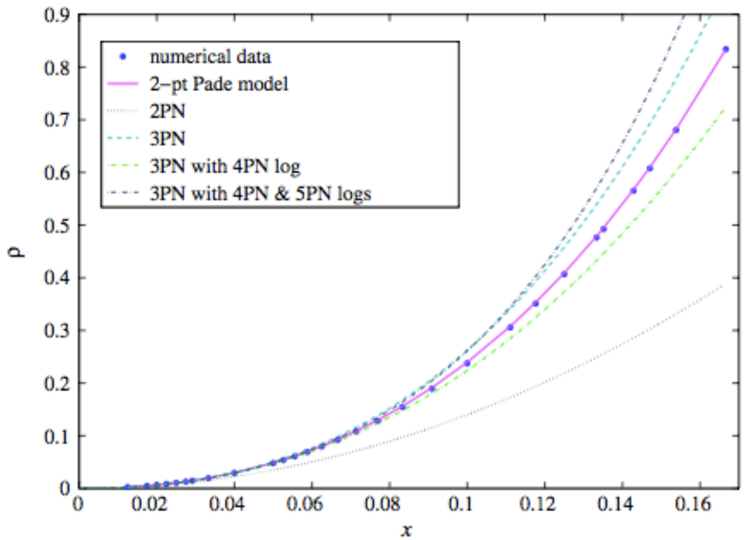
Precession function of small-eccentricity orbits: $\left(\frac{\omega_r}{\Omega}\right)^2 = 1 - 6x + \nu\rho(x) + O(\nu^2)$

$$\rho(x) = 4x\left(1 - \frac{1-2x}{\sqrt{1-3x}}\right) + a_1(x) + xa_1'(x) + \frac{1}{2}x(1-2x)a_1''(x) + (1-6x)\bar{d}_1(x)$$

Gauge-invariant scattering angle: $\chi^{\rm scattering} = \chi(\mathcal{E}, \mathcal{J})$

Angular momentum and whirl frequency of zero-binding zoom-whirl orbit (**see Colleoni's talk**)

First SF computation of a strong-field EOB potential Barack-Damour-Sago 2010

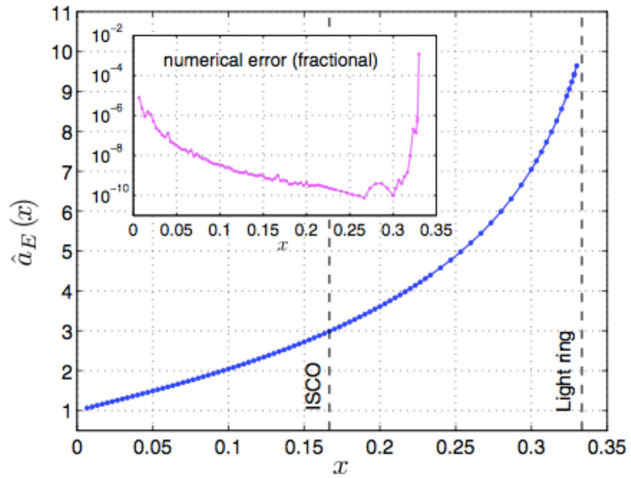


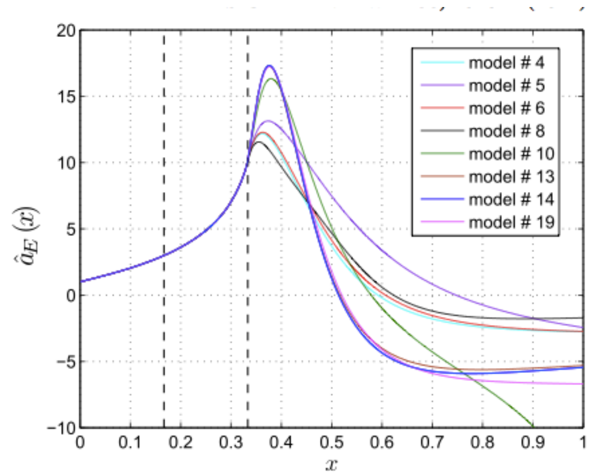
OTHER SF-DERIVED EOB INFORMATION

Detweiler's redshift 08 + First Law (LeTiec-Blanchet-Whiting 12, Blanchet-Buonanno-LeTiec 13, LeTiec 15):

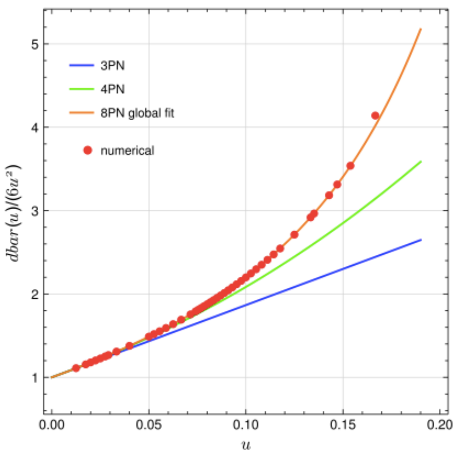
Barausse-Buonanno-LeTiec , Akcay-Barack-Damour-Sago, Bini-Damour, LeTiec, Bini-Damour-Geralico, Hopper-Kavanagh-Ottewill, Akcay-vandeMeent,…

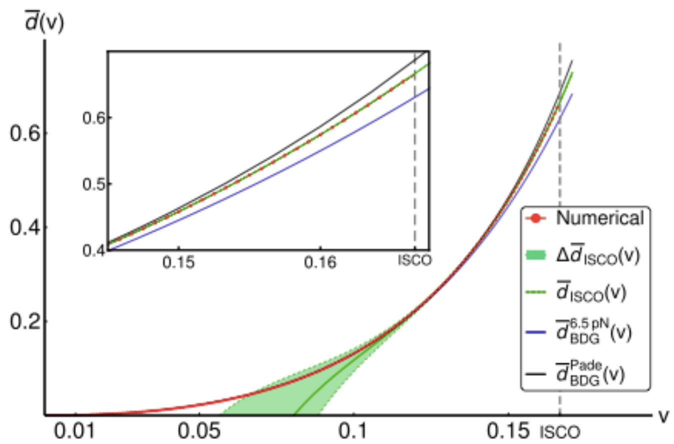
doubly rescaled a_1(u)



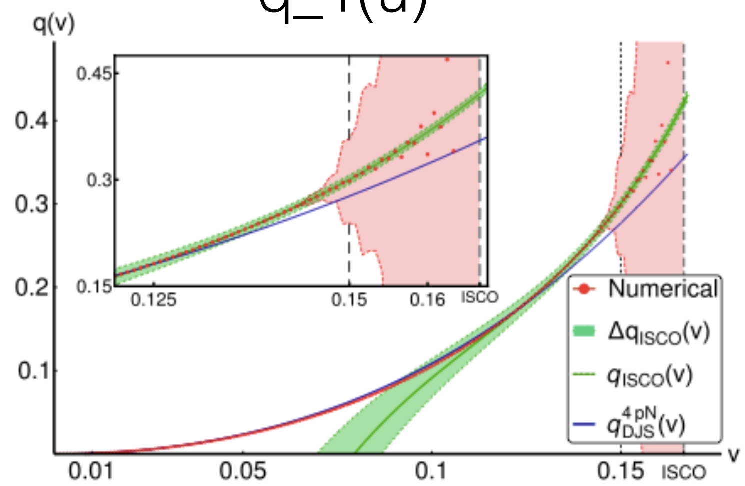


d_1(u)





q_1(u)



GSF : ANALYTICAL HIGH-PN RESULTS

Bini-Damour 15

$$
\begin{aligned}
a_{10}^{c} = {} & \frac{18605478842060273}{7079830758000}\ln(2) - \frac{1619008}{405}\zeta(3) - \frac{21339873214728097}{1011404394000}\gamma \\
& + \frac{27101981341}{100663296}\pi^6 - \frac{6236861670873}{125565440}\ln(3) + \frac{360126}{49}\ln(2)\ln(3) + \frac{180063}{49}\ln(3)^2 \\
& - \frac{121494974752}{9823275}\ln(2)^2 - \frac{24229836023352153}{549755813888}\pi^4 + \frac{1115369140625}{124540416}\ln(5) + \frac{96889}{2779} \\
& + \frac{754370143706233186232999}{186907532011200000} - \frac{60648244288}{9823275}\ln(2)\gamma + \frac{200706848}{280665}\gamma^2 \\
& + \frac{11980569677139}{2306867200}\pi^2 + \frac{360126}{49}\gamma\ln(3), \\
a_{10}^{\ln} = {} & -\frac{21275143333512097}{2022808788000} + \frac{200706848}{280665}\gamma - \frac{30324122144}{9823275}\ln(2) + \frac{180063}{49}\ln(3), \\
a_{10}^{\ln^2} = {} & \frac{50176712}{280665}, \\
a_{10.5}^{c} = {} & -\frac{185665618769828101}{24473489040000}\pi + \frac{377443508}{77175}\ln(2)\pi + \frac{2414166668}{1157625}\pi\gamma - \frac{5846788}{11025}\pi^3 - \frac{24}{} \\
a_{10.5}^{\ln} = {} & \frac{1207083334}{1157625}\pi.
\end{aligned}
$$

Kavanagh et al 15

$$
\begin{aligned}
c_{15} = {} & -\frac{2069543450583769619340376724}{325477442086506084375}\zeta(3) + \frac{6519502629824500793 6}{22370298575625}\gamma\zeta(3) - \frac{5049442304}{25725}\gamma^2\zeta(3) + \frac{1262360576}{15435}\pi^2\zeta(3) \\
& + \frac{171722752}{441}\zeta(3)^2 + \frac{1613866959570176}{496621125}\zeta(5) - \frac{343445504}{441}\gamma\zeta(5) - \frac{146997248}{105}\zeta(7) + \frac{56314978304}{385875}\zeta(3)\log^2(2) \\
& - \frac{106445664}{343}\zeta(3)\log^2(3) + \frac{15167099824484979769 6}{22370298575625}\zeta(3)\log(2) - \frac{190336581632}{1157625}\gamma\zeta(3)\log(2) \\
& + \frac{28863591064624341}{4909804900}\zeta(3)\log(3) - \frac{212891328}{343}\gamma\zeta(3)\log(3) - \frac{212891328}{343}\zeta(3)\log(2)\log(3) - \frac{77186767578125}{19876428}\zeta(3)\log \\
& - \frac{2039263232}{3675}\zeta(5)\log(2) - \frac{49128768}{49}\zeta(5)\log(3) + \frac{29826742751501839701973659217528941950139153944429084 9}{65876122225446532261424684050319173195312 50} \\
& - \frac{6807661768453637768313286948060329087501419}{7043109481248037225626077295440625 00}\gamma + \frac{15983469444126032478310062898293 88}{52617171503867703359189062 5}\gamma^2 - \frac{1007647146215971027644}{335890033113009375} \\
& + \frac{461219496448}{72930375}\gamma^4 - \frac{28338275082077591587855063450276303790065762907243197}{999703155845143418115744045792755712000000}\pi^2 + \frac{2519117865539927569110 4}{67178006622601875}\gamma\pi^2 \\
& - \frac{230609748224}{14586075}\gamma^2\pi^2 + \frac{10548032335775722689471378776039118077624803624 1}{304245354831316028025099055320268800000}\pi^4 + \frac{1262360576}{385875}\gamma\pi^4 \\
& - \frac{620847283961296697269145713114 3}{266930151354100246118400}\pi^6 + \frac{357317878192092911828132 9}{151996487423754240}\pi^8 - \frac{10136323685888}{72930375}\log^4(2) + \frac{38438712}{2401}\log^4(3) \\
& - \frac{1778960861264826796478 72}{549638236003106 25}\log^3(2) - \frac{89686013106176}{364651875}\gamma\log^3(2) + \frac{153754848}{2401}\log^3(2)\log(3) \\
& - \frac{1314638453227902691 23}{245735735245000}\log^3(3) + \frac{153754848}{2401}\gamma\log^3(3) + \frac{153754848}{2401}\log(2)\log^3(3) + \frac{119330742675781 25}{51161925672}\log^3(5) \\
& + \frac{38782586741666289745954206352002 04}{18942181741392373209308062 5}\log^2(2) - \frac{34408563799146016921511 68}{1007670099339028125}\gamma\log^2(2) - \frac{16582891400192}{121550625}\gamma^2\log^2(2) \\
& + \frac{4145722850048}{72930375}\pi^2\log^2(2) - \frac{5236971633734839056 09}{245735735245000}\log^2(2)\log(3) + \frac{461264544}{2401}\gamma\log^2(2)\log(3) \\
& + \frac{45454535766189065888302299261759}{65697282267898830348800 00}\log^2(3) - \frac{394391535968370807369}{245735735245000}\gamma\log^2(3) + \frac{230632272}{2401}\gamma^2\log^2(3) \\
& - \frac{96096780}{2401}\pi^2\log^2(3) - \frac{437493411770075173449}{245735735245000}\log(2)\log^2(3) + \frac{461264544}{2401}\gamma\log(2)\log^2(3) \\
& + \frac{230632272}{2401}\log^2(2)\log^2(3) + \frac{11933074267578125}{17053975224}\log^2(2)\log(5) - \frac{2505842696993145943705498046875}{402136320895332222431232}\log^2(5) \\
& + \frac{11933074267578125}{17053975224}\gamma\log^2(5) + \frac{11933074267578125}{17053975224}\log(2)\log^2(5) + \frac{479295083164704151420102 51}{5646463517021184000 0}\log^2(7) \\
& - \frac{18163606721689522042153774768525369973449465 9}{63387985331232335030634695658965625 00}\log(2) + \frac{7420366215521910854379953165301013 6}{4735545435348093302327015625}\gamma\log(2) \\
& - \frac{1482169326522492515499392}{1007670099339028125}\gamma^2\log(2) - \frac{4905667647488}{364651875}\gamma^3\log(2) + \frac{37122811549066766845116 8}{604602059603416875}\pi^2\log(2) \\
& + \frac{1226416911872}{72930375}\gamma\pi^2\log(2) + \frac{23792072704}{17364375}\pi^4\log(2) - \frac{414115837539718030238709512493585574772 7}{1082666315962744888801986560000 00}\log(3) \\
& + \frac{94593580011315754543320552762 39}{691550339662092951040000}\gamma\log(3) - \frac{394391535968370807369}{245735735245000}\gamma^2\log(3) + \frac{153754848}{2401}\gamma^3\log(3) \\
& + \frac{131463845322790269123}{196588588196000}\pi^2\log(3) - \frac{192193560}{2401}\gamma\pi^2\log(3) + \frac{8870472}{1715}\pi^4\log(3) \\
& + \frac{214411501060211389845962927148381}{1313945645357976606976 0000}\log(2)\log(3) - \frac{437493411770075173449}{122867867622500}\gamma\log(2)\log(3) \\
& + \frac{461264544}{2401}\gamma^2\log(2)\log(3) - \frac{192193560}{2401}\pi^2\log(2)\log(3) + \frac{97861294850170985327709557611886523437 5}{17942749191956127021132384903168}\log(5) \\
& - \frac{2505842696993145943705498046875}{201068160447666111215616}\gamma\log(5) + \frac{11933074267578125}{17053975224}\gamma^2\log(5) - \frac{59665371337890625}{204647702688}\pi^2\log(5) \\
& - \frac{2505842696993145943705498046875}{201068160447666111215616}\log(2)\log(5) + \frac{11933074267578125}{8526987612}\gamma\log(2)\log(5) \\
& - \frac{58580061737923089156651130139146480 81}{3239191932075128029777920000 00}\log(7) + \frac{479295083164704151420102 51}{28232317585105920000}\gamma\log(7) \\
& + \frac{479295083164704151420102 51}{28232317585105920000}\log(2)\log(7) + \frac{740024994425816010121 1}{65676344832000000}\log(11),
\end{aligned}
$$

SF CONFIRMATIONS OF ADM 4PN DYNAMICS

Damour-Jaranowski-Schaefer 2016 using several bridges (EOB, periastron precession, First Law BBH Dynamics [LeTiec-Blanchet-Whiting 2012], order-reduction) and recent SF results [vandeMeent-Shah2015; Bini-Damour-Geralico2016; Hopper-Kavanagh-Ottewill2016; Akcay-vandeMeent2016; vandeMeent2016;…

$$a_5^c = \frac{2275\pi^2}{512} - \frac{4237}{60} + \frac{128}{5}\gamma_E + \frac{256}{5}\ln 2, \tag{2.15a}$$

$$a_5^{\ln} = \frac{64}{5}, \tag{2.15b}$$

$$\bar{d}_4^c = -\frac{533}{45} - \frac{23761\pi^2}{1536} + \frac{1184}{15}\gamma_E - \frac{6496}{15}\ln 2 + \frac{2916}{5}\ln 3, \tag{2.15c}$$

$$\bar{d}_4^{\ln} = \frac{592}{15}, \tag{2.15d}$$

$$q_{4,3} = -\frac{5308}{15} + \frac{496256}{45}\ln 2 - \frac{33048}{5}\ln 3, \tag{2.15e}$$

$$q_{6,2} = -\frac{827}{3} - \frac{2358912}{25}\ln 2 + \frac{1399437}{50}\ln 3 + \frac{390625}{18}\ln 5. \tag{2.15f}$$

$$\rho_4^{c,\mathrm{DJS}} = \frac{58265}{1536}\pi^2 - \frac{215729}{180} + \frac{5024}{15}\gamma_E + \frac{1184}{15}\ln 2 + \frac{2916}{5}\ln 3. = 64.6405647571193\ldots\ldots$$

$$\rho_4^{c,\mathrm{num}[35]} = 64.640566(2), \quad \text{(van de Meent 16)}$$

$$\Delta\rho_4^c \equiv \rho_4^{c,\mathrm{num}[35]} - \rho_4^{c,\mathrm{DJS}} = (1 \pm 2) \times 10^{-6}.$$

incompatible with

Bernard-Blanchet-Bohe-Faye-Marsat 16.

$$\delta^{\mathrm{B^3FM}}\rho_4^c = \frac{44026}{315} \approx 139.7650794.$$

SF-DETERMINED SPIN-DEPENDENT COUPLING FUNCTIONS

Bini-Damour-Geralico 2016: analytical and SF-numerical-extracted results

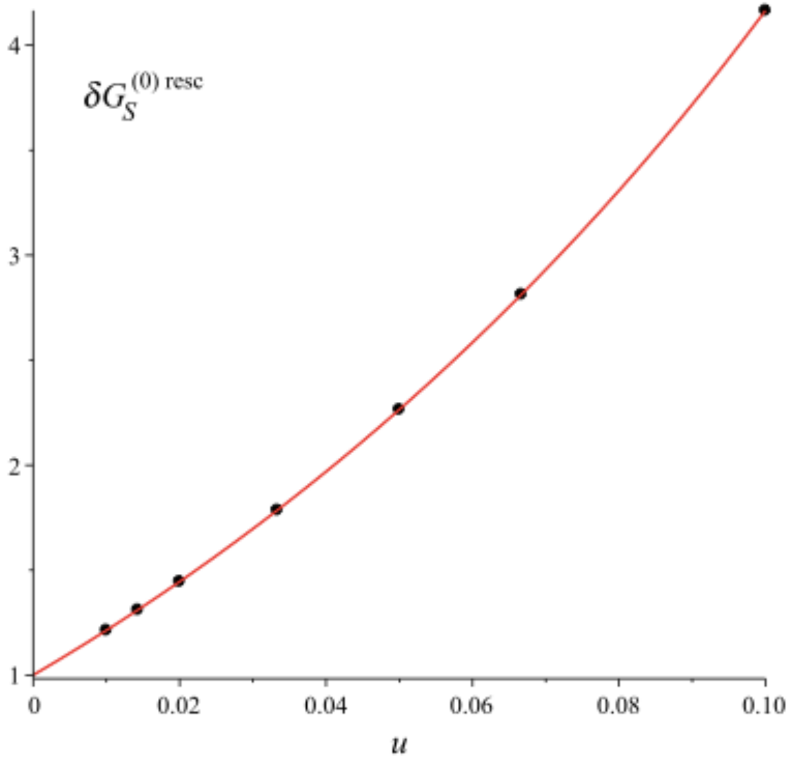
$$A(r,M,\nu,S_1,S_2) = A^{\rm Kerr}(r,M,a_1+a_2) + \nu(A^{(0)}_{1SF}(u) + \hat{a}_2^2 f_A^{(0)}(u) + \ldots) + \ldots$$

$$G_S(r,M,\nu,S_1,S_2) = G_S^{\rm Kerr}(r,M,a_1+a_2) + \nu({G_S^{(0)}}_{1SF}(u) + \hat{a}_2^2 G_S^{(2)} + \ldots) + \ldots$$

$$\begin{aligned}
\nu^{-1}\delta G_S^{(0)\,\rm resc}(u) &= 1 + \frac{102}{5}u + \left(\frac{80399}{720} - \frac{241}{120}\pi^2\right)u^2 + \left(\frac{12015517}{18000} - \frac{62041}{960}\pi^2 + \frac{3712}{15}\ln(2) + \frac{9344}{75}\gamma + \frac{4672}{75}\ln u\right)u^3 \\
&+ \left(-\frac{437083}{256}\pi^2 - \frac{122848}{105}\ln(2) + \frac{122119110037}{7056000} + \frac{7776}{35}\ln(3) - \frac{250336}{525}\gamma - \frac{24496}{105}\ln(u)\right)u^4 \\
&+ \frac{1372352}{7875}\pi u^{9/2} + \left(\frac{753139951463}{17860500} - \frac{6377586259}{1105920}\pi^2 + \frac{1210624}{2835}\ln(2) - \frac{67068}{35}\ln(3) - \frac{13977664}{14175}\gamma \right. \\
&\left. + \frac{8776579}{40960}\pi^4 - \frac{6807392}{14175}\ln(u)\right)u^5 - \frac{18186214}{23625}\pi u^{11/2} \\
&+ \left[-\frac{350464}{225}\gamma^2 + \frac{75337409381}{62914560}\pi^4 + \frac{1847725444}{111375}\gamma - \frac{49010176}{7875}\ln(2)\gamma + \frac{34368219}{7700}\ln(3) \right. \\
&+ \frac{556745811545134537}{18441561600000} - \frac{7009035336469}{442368000}\pi^2 + \frac{204853984012}{9095625}\ln(2) + \frac{390625}{396}\ln(5) + \frac{39424}{15}\zeta(3) \\
&\left. - \frac{48982784}{7875}\ln(2)^2 + \left(\frac{6536679614}{779625} - \frac{24505088}{7875}\ln(2) - \frac{350464}{225}\gamma\right)\ln(u) - \frac{87616}{225}\ln(u)^2\right]u^6 \\
&- \frac{33472011779}{49116375}\pi u^{13/2} + O(u^7 \ln u),
\end{aligned} \qquad (4.3)$$

SF-DETERMINED SPIN-DEPENDENT COUPLING FUNCTIONS

SF-numerical data extracted from Shah-Friedman-Keidl 12 redshift data around Kerr (corrected) compared to analytical BDG 16 results



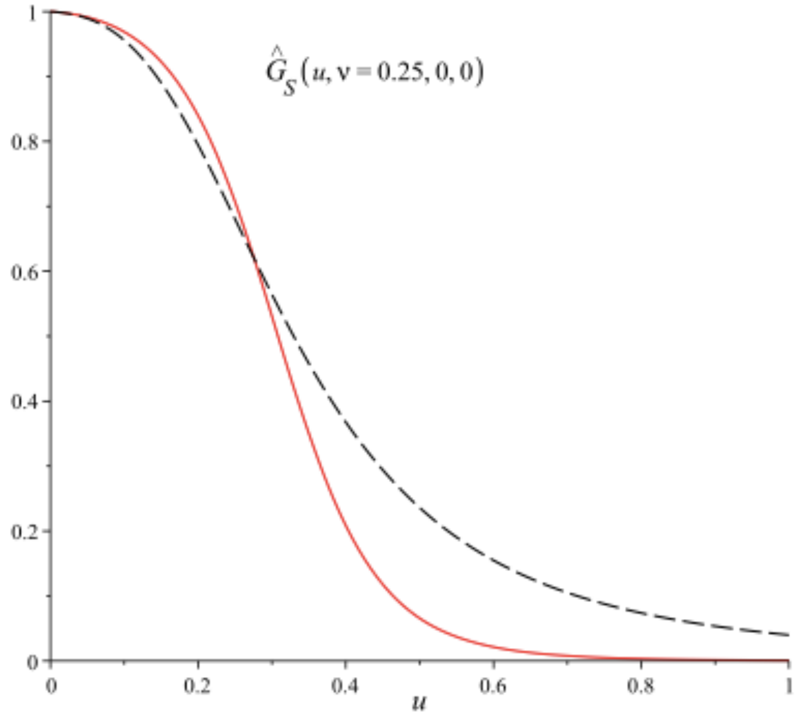


FIG. 5 (color online). The quantity $\hat{G}_S(u, \nu, 0, 0)$ used in Damour-Nagar (dashed curve) [39] as well as the present determination (solid curve) are compared in the case $\nu = 0.25$.

SF-DETERMINED TIDAL COUPLING FUNCTIONS

2PN knowledge (Bini-Damour-Faye 12)

$$\hat{A}_1^{(2^+)}(u;X_1) = 1 + \frac{3u^2}{1-3u} + \frac{5}{2}X_1 u + \left(\frac{1}{8}X_1 + \frac{337}{28}X_1^2\right)u^2 + O_{X_1}(u^3).$$

High-order analytical SF knowledge Bini-Damour 14

$$\begin{aligned}
\hat{A}_1^{(2^+)1\mathrm{SF}}(u) = {} & \frac{5}{2}u + \frac{1}{8}u^2 + \left(-\frac{3487}{16} + \frac{1905}{256}\pi^2\right)u^3 + \left(-\frac{2816}{5}\ln(2) - \frac{1408}{5}\gamma + \frac{4114603}{9600} - \frac{25337}{1024}\pi^2 - \frac{704}{5}\ln u\right)u^4 \\
& + \left(\frac{1146014221}{403200} - \frac{10387277}{24576}\pi^2 + \frac{66056}{105}\ln u + \frac{132112}{105}\gamma + 3152\ln 2 - \frac{4374}{7}\ln 3\right)u^5 - \frac{6848}{21}\pi u^{11/2} \\
& + \left(\frac{30246655583}{2903040} + \frac{34840}{63}\ln(u) + \frac{69680}{63}\gamma - \frac{5150464}{945}\ln(2) + 6075\ln(3)\right. \\
& \left. - \frac{2494654027}{786432}\pi^2 + \frac{76231071}{1048576}\pi^4\right)u^6 + \frac{11417267}{7350}\pi u^{13/2} \\
& + \left(\frac{9943210070208659}{55883520000} - \frac{19182623242}{779625)}\ln(2) - \frac{13278187073}{779625}\ln(u) - \frac{37249370460407}{2831155200}\pi^2\right. \\
& - \frac{26556374146}{779625}\gamma - \frac{49369095}{2464}\ln(3) - \frac{19782291875}{67108864}\pi^4 + \frac{438272}{75}\ln(2)\ln(u) + \frac{219136}{75}\ln(u)\gamma \\
& \left. - \frac{28672}{5}\zeta(3) - \frac{37109375}{9504}\ln(5) + \frac{876544}{75}\ln^2(2) + \frac{219136}{75}\gamma^2 + \frac{876544}{75}\gamma\ln(2) + \frac{54784}{75}\ln^2(u)\right)u^7 \\
& + \frac{283918559}{485100}\pi u^{15/2} + O_{\ln}(u^8).
\end{aligned} \tag{6.23}$$

SF-DETERMINED TIDAL KNOWLEDGE

BD14: combining PN, SF-analytical, SF-numerical (Dolan-Nolan-Ottewill-Warburton-Wardell14) and EOB-light-ring info

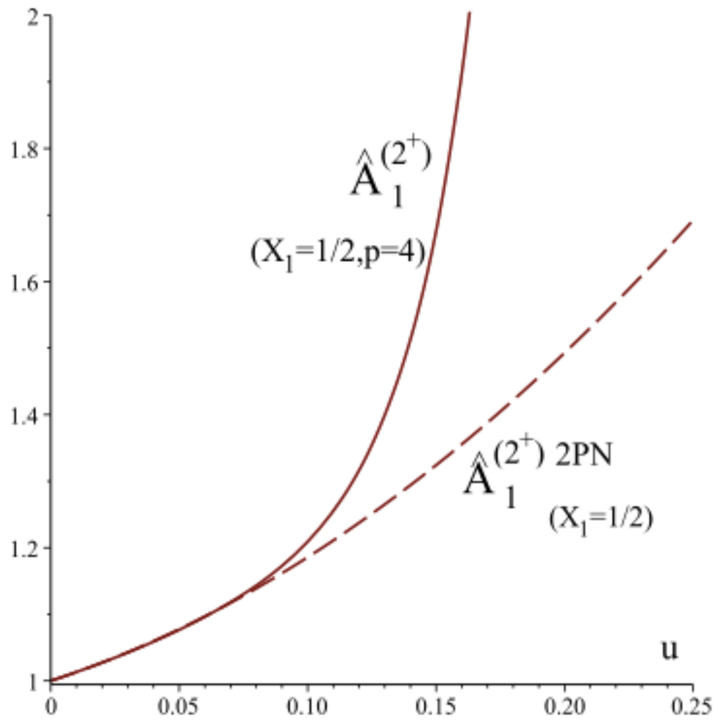
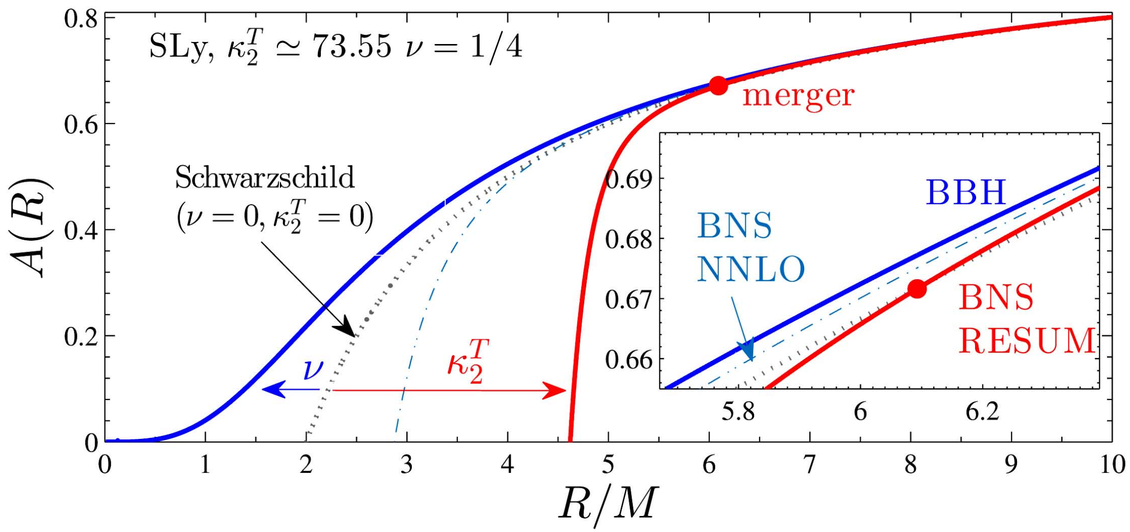


FIG. 5 (color online). The full quadrupolar-electric tidal factor $\hat{A}_1^{(2^+)}(u; X_1)$, Eq. (7.33), is plotted (solid line) as a function of the EOB variable u, for the choice of parameters $p = 4$ and $X_1 = \frac{1}{2}$. For comparison, we plot, as a dashed line, its standardly used 2PN-accurate approximation $\hat{A}_1^{(2^+)2\mathrm{PN}}(u; X_1) = 1 + \alpha_1^{(2^+)}(X_1)u + \alpha_2^{(2^+)}(X_1)u^2$.

$$\hat{A}_1^{(2^+)}(u; X_1) = 1 + \frac{3u^2}{1-3u} + X_1 \frac{\tilde{A}_1^{(2^+)1\mathrm{SF}}(u)}{(1-3u)^{7/2}} + X_1^2 \frac{\tilde{A}_1^{(2^+)2\mathrm{SF}}(u)}{(1-3u)^p} + O(X_1^3 u^3),$$

Used for NR-comparisons in Bernuzzi et al 15



FUTURE PROSPECTS EOB+SF

2SF approximation $$A(u) = 1 - 2u + \nu a_1(u) + \nu^2 a_2(u) + O(\nu^3)$$

Bini-Damour 16

$$z_{2\mathrm{SF}}^{\mathrm{PN}}(y) = z_{2\mathrm{SF}}^{\mathrm{known}}(y) + z_{2\mathrm{SF}}^{a_2\text{-unknown}}(y), \qquad \hat{z}_{2\mathrm{SF}}^{\mathrm{PN}}(y) = \hat{z}_{2\mathrm{SF}}^{\mathrm{known}}(y) + \hat{z}_{2\mathrm{SF}}^{a_2\text{-unknown}}(y),$$
$$U_{2\mathrm{SF}}^{\mathrm{PN}}(y) = U_{2\mathrm{SF}}^{\mathrm{known}}(y) + U_{2\mathrm{SF}}^{a_2\text{-unknown}}(y), \qquad \hat{U}_{2\mathrm{SF}}^{\mathrm{PN}}(y) = \hat{U}_{2\mathrm{SF}}^{\mathrm{known}}(y) + \hat{U}_{2\mathrm{SF}}^{a_2\text{-unknown}}(y),$$

namely

$$\begin{aligned} z_{2\mathrm{SF}}^{\mathrm{known}}(y) = {} & -y + y^2 - \frac{29}{8}y^3 + \left(-\frac{74}{3} + \frac{41}{64}\pi^2\right)y^4 + \left(\frac{64}{5}\gamma + \frac{128}{5}\ln(2) + \frac{4899}{1024}\pi^2 + \frac{32}{5}\ln(y) - \frac{75107}{640}\right)y^5 \\ & + \left(-\frac{6556}{35}\gamma - \frac{14972}{35}\ln(2) + \frac{729}{14}\ln(3) + \frac{232221}{2048}\pi^2 - \frac{958}{7}\ln(y) - \frac{66534539}{67200}\right)y^6 + \frac{27392}{525}\pi y^{13/2} \\ & + \left(\frac{351398}{2835}\gamma + \frac{3312926}{2835}\ln(2) - \frac{21627}{28}\ln(3) + \frac{3018779419}{3538944}\pi^2 - \frac{12283021}{524288}\pi^4 \right. \\ & \left. - \frac{8009}{2835}\ln(y) - \frac{50685282659}{14515200}\right)y^7 - \frac{1345759}{3675}\pi y^{15/2} \end{aligned}$$

$$z_{2\mathrm{SF}}^{a_2\text{-unknown}}(y) = \frac{3}{2}a_6' y^6 + \left(\frac{9}{4}a_6' + \frac{3}{2}a_7' + \frac{3}{2}a_7'^{\ln}\ln(y)\right)y^7 + \frac{3}{2}a_{7.5}' y^{15/2},$$

$$(m_1 + m_2)\Omega_{\mathrm{LSO}}(\nu) = 6^{-3/2}[1 + \nu C_\Omega + \nu^2 D_\Omega] + O(\nu^3),$$

$$D_\Omega = 4.2858457(7) + 0.0021541 a_6' - 0.0007386 a_7'^{\ln} + 0.0004930 a_7' + 0.0002319 a_{7.5}'. \qquad (4.17)$$

EOB[SF] for LISA

Completing EOB conservative dynamics and radiation reaction by both non-circular PN info (a la Bini-Damour 12) and by SF knowledge