Completion of metric reconstruction

for a particle orbiting a Kerr black hole

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1. Motivation & statement of the problem

1.1 Recent developments

Completion Problem (aka "$\ell = 0, 1$" problem) occupied Capra community since early 2000s. Must be addressed to take full advantage of 3 recent developments:

- **Progress on metric reconstruction** from Weyl curvature scalars, for an orbiting particle [Ori (2003); Keidl, Friedman, Shah et al (2007-2012); van de Meent & Shah (2015)]
- **Formulation of self-force and motion from a reconstructed metric**, with a practical mode-sum formula [Pound, Merlin & LB (2014)]
- **Advances in semi-analytic methods** for solving the Teukolsky equation, based on the Mano-Suzuki-Takasugi (MST) approach [Hughes; Shah et al; van de Meent & Shah; Bini & Damour; Kavanagh et al;. . .]

1. Motivation & statement of the problem

1.2 Progress on metric reconstruction

- **Reconstruction in vacuum** [Chrzanowski 1975; Cohen & Kegeles 1979 (CCK); Wald 1973, 1978; Stewart 1979; Lousto & Whiting 2002]

  Let h_{ab} be a vacuum metric perturbation of Kerr and ψ_4 associated Weyl scalar. Then, given any Ψ satisfying

  $$\Box_{\rm Teukolsky}\Psi = 0, \qquad D^4\Psi = \psi_4,$$

  h_{ab} may be reproduced ("reconstructed") using

  $$h_{ab}^{(\rm rec)} = \mathcal{D}^{(2)}_{CCK}\Psi,$$

  to within a gauge perturbation and a linear combination of 4 "trivial" homogeneous solutions:

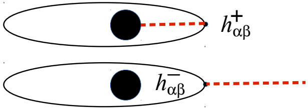
  $$c_1 h_{ab}^{\delta M} + c_2 h_{ab}^{\delta J} + c_3 h_{ab}^{\rm C\,metric} + c_4 h_{ab}^{\rm KerrNUT}.$$

  Completion: the task of determining c_n.

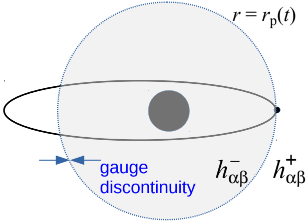
1. Motivation & statement of the problem

1.2 Progress on metric reconstruction (cont'd)

- **In presence of matter sources**, CCK procedure fails to yield a valid solution even in vacuum away from sources [LB & Ori 2001; Price & Whiting 2007]
- Reconstruction for bound orbits, with string-like gauge singularities [Ori 2003]
- Reconstruction for circular equatorial orbits, with gauge discontinuity on a sphere [Keidl, Friedman etal (2007-2012)]
- Extension to any bound equatorial orbit [van de Meent & Shah (2015)]



Ori's "half-string" solutions



Friedman's "no-string" solution

1. Motivation & statement of the problem

1.3 Self-force from a reconstructed metric [Pound, Merlin & LB (2014)]

Two alternative mode-sum schemes:

$$F_\alpha^{\pm} = \sum_{\ell=0}^{\infty} \left[(F_\alpha)_{\pm}^{\ell} - (\ell + 1/2) A_\alpha^{\pm} - B_\alpha \right] - \delta D_\alpha^{\pm}$$

in a local Lorenz deformation of either of the half-string radiation gauges

$$F_\alpha = \sum_{\ell=0}^{\infty} \left[\frac{1}{2} (F_\alpha)_{+}^{\ell} + \frac{1}{2} (F_\alpha)_{-}^{\ell} - B_\alpha \right]$$

in a no-string radiation gauge

- $A_\alpha^{\pm}$ and B_α are Lorenz-gauge regularization parameters.
- Values of $\delta D_\alpha^{\pm}$ depend on off-particle extension used to define $(F_\alpha)_{\pm}^{\ell}$.
- $(F_\alpha)_{\pm}^{\ell}$ are constructed from the modes of the **completed** perturbations $h_{\alpha\beta}^{\pm}$.

1. Motivation & statement of the problem

1.4 Completion for a particle source

- Need to determine $c^{\pm}$ in

$$h_{ab}^{(\mathrm{comp})\pm} := c_1^{\pm} h_{ab}^{\delta M} + c_2^{\pm} h_{ab}^{\delta J} + c_3^{\pm} h_{ab}^{\mathrm{C\,metric}} + c_4^{\pm} h_{ab}^{\mathrm{KerrNUT}}$$

- $c_3^{\pm} = 0 = c_4^{\pm}$ from regularity [Keidl, Shah, Friedman, Kim & Price (2010)]
- c_1^{+} and c_2^{+} are readily determined from $r \to \infty$ asymptotics, given total ADM mass and angular-momentum of system.
- So, the task is to determine the pair of constants c_1^{-} and c_2^{-}, or, equivalently

$$[\mathcal{E}] := c_1^{+} - c_1^{-}, \qquad [\mathcal{J}] := c_2^{+} - c_2^{-}$$

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Why not just use F_{α}^{+}?

- Calculation of δD_{α}^{+} is hard and subtle
- Wish to have control over internal perturbation and mass of black hole

2. Earlier attempts at the problem

- Larry Price (PhD thesis, 2007): Demand that $h_{ab}^{\mathrm{comp}+}$ and $h_{ab}^{\mathrm{comp}-}$ match smoothly on $\bar{\mathcal{S}}$ up to a gauge transformation. Makes sense under the assumption that $h_{ab}^{\mathrm{rec}+}$ and $h_{ab}^{\mathrm{rec}-}$ can be matched smoothly (up to gauge). Applied for Schwarzschild (circular orbits) only.

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- Dolan & LB (2013), following Abbot & Desser (1982): Quasi-local conserved integrals (requiring only *background* symmetries) may be used to determine mass and AM content of $h_{ab}^{\rm rec\pm}$. Can be used to easily fix completion piece in Schwarzschild, but hard to apply in Kerr.

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- Sano & Tagoshi (2014): Considered a rotating circular mass ring. Require continuity on $\bar{\mathcal{S}}$ of the metric perturbation and of ψ_1, ψ_2, ψ_3. However, allow for singularities in the equatorial plane in/outside the ring, so uniqueness of completion unclear.

3. Our strategy

Determine $[\mathcal{E}]$ and $[\mathcal{J}]$ by demanding that certain gauge-invariant fields constructed from the completed perturbation are continuous on $\bar{\mathcal{S}}$:

$$\mathcal{I}(h_{ab}^{\rm rec+} + h_{ab}^{\rm comp+})\Big|_{\bar{S}} = \mathcal{I}(h_{ab}^{\rm rec-} + h_{ab}^{\rm comp-})\Big|_{\bar{S}}$$

(Need two such conditions: either two invariants evaluated at a certain θ, or a single invariant evaluated at two values of θ.)

The resulting completed perturbation (unlike $h^{\rm rec}$) should be a vacuum solution of the linearized EFE anywhere off the particle. But it is not necessarily smooth (or even continuous) across $\bar{\mathcal{S}}$.

Here we do not take on the more ambitious task of "gauge smoothing", which is required for some applications (cf. Maarten van de Meent's talk).

4. Auxiliary gauge-invariant fields

- Introduce a reference gauge, in which $\delta\psi_2 = 0$:

$$\delta\psi_2^{\rm orig} \to \delta\psi_2^{\rm ref} = \delta\psi_2^{\rm orig} - \xi^\alpha \psi_{2,\alpha}^{(0)} \equiv 0$$

$$\Rightarrow \quad \xi^r = \frac{{\rm Re}(\varrho^{-4}\delta\psi_2^{\rm orig})}{3M}, \qquad \xi^\theta = \frac{{\rm Im}(\varrho^{-4}\delta\psi_2^{\rm orig})}{3Ma\sin\theta},$$

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- The $\{rr, r\theta, \theta\theta\}$ components of the perturbation in the reference gauge,

$$h_{\alpha\beta}^{\rm ref} = h_{\alpha\beta}^{\rm orig} - 2\xi_{\alpha,\beta} + 2\Gamma_{\alpha\beta}^{(0)\gamma}\xi_\gamma,$$

  are completely determined from $\{\xi^r, \xi^\theta\}$.

- So take $\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3\} := \left\{h_{rr}^{\rm ref}, h_{\theta\theta}^{\rm ref}, h_{r\theta}^{\rm ref}\right\}$ as our invariant fields. Think of each of these as a (3rd-order) differential operator on $h_{\alpha\beta}^{\rm orig}$.

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- For $a = 0$ take instead $\{\mathcal{I}_1, \mathcal{I}_2\} := \left\{h_{rr}^{\rm ref}, \mathrm{Im}(\delta\psi_2^{\rm orig})\right\}$.

5. Determination of the completion amplitudes:
5.1 Circular equatorial orbits in Kerr

Step 1: Write the Stationary & axisymmetric part of ψ_4 as a sum over harmonics:

$$\psi_4^{\mathrm{SAS}} = \varrho^4 \sum_{\ell=2}^{\infty} R_\ell(r)\, {}_{-2}Y_{\ell 0}(\theta),$$

Step 2: Solve the inhomogeneous Teukolsky equation with regular BCs, to obtain

$$R_\ell(r;r_0) = C_\ell^+(r_0)R_\ell^+(r)\theta(r-r_0) + C_\ell^-(r_0)R_\ell^-(r)\theta(r_0-r) + C_\ell^\delta(r_0)\delta(r-r_0),$$

where $R_\ell^\pm$ are suitable homogeneous solutions, and $C_\ell^\pm = \sum_{n,j=0}^{2} \alpha_{nj}(r_0)\, \left.\frac{d^n R_\ell^\mp}{dr^n}\right|_{r_0} \left.\frac{d^j\, {}_2Y_{\ell 0}}{d\theta^j}\right|_{\theta_0}$.

Step 3: Solve the "inversion" equation for the Hertz potential,

$$\bar{\eth}^4 \bar{\Psi}^\pm = 8\varrho^{-4}\psi_4^{\mathrm{SAS}\pm},$$

on either sides of $\bar{\mathcal{S}}$, to obtain

$$\bar{\Psi}^\pm = \sum_{\ell=2}^{\infty} \frac{8(\ell-2)!}{(\ell+2)!} C_\ell^\pm(r_0) R_\ell^\pm(r)\, {}_{+2}Y_{\ell 0}(\theta).$$

5. Determination of the completion amplitudes:

5.1 Circular equatorial orbits in Kerr (cont'd)

Step 4: Obtain (reconstructed bits of) metric components and $\delta\psi_2$:

$$
\begin{aligned}
h_{rr}^{\rm rec\pm} &= -{\rm Re}\,\frac{1}{\Delta^2\bar\varrho^4}\,\bar\eth_1\left(\bar\varrho^2\bar\eth_2\bar\Psi^\pm\right), \\
h_{\theta\theta}^{\rm rec\pm} &= -{\rm Re}\,\frac{1}{\bar\varrho^4}\partial_r\left(\bar\varrho^2\partial_r\bar\Psi^\pm\right). \\
\delta\psi_2^{\rm rec\pm} &= \frac{1}{4}\partial_r^2\left(\varrho^2\bar\eth_1\bar\eth_2\bar\Psi^\pm\right) - \frac{\varrho_{,\theta}}{\varrho}\partial_r\left[\varrho\partial_r\left(\varrho\,\bar\eth_2\bar\Psi^\pm\right)\right] + \frac{3}{2}\varrho_{,\theta}\partial_r\left(\varrho_{,\theta}\partial_r\bar\Psi^\pm\right).
\end{aligned}
$$

Step 5: Construct (reconstructed bits of) gauge-invariant fields:

$$
\mathcal{I}_n^{\rm rec\pm} = \sum_{\ell=2}^{\infty}\sum_{j=0}^{3}\sum_{k=0}^{3}\frac{(\ell-2)!}{(\ell+2)!}f_{njk}(r,\theta)C_\ell^\mp(r_0)R_\ell^{\pm(k)}(r)\,{}_2Y_{\ell 0}^{(j)}(\theta)
$$

Step 5: Obtain jump across $\bar{\mathcal{S}}$; simplify using Wronskian$=\Delta$:

$$
\left[\mathcal{I}_n^{\rm rec}\right](\theta;r_0) = \sum_{j=0}^{3}\sum_{i=0}^{2}\sum_{k=0}^{3} h_{njik}(\theta;r_0)\sum_{\ell=2}^{\infty}\Lambda_{\ell k}\,{}_2Y_\ell^{(j)}(\theta)\,{}_2Y_\ell^{(i)}(\theta_0),
$$

where $\Lambda_{\ell k} := \left\{1, \frac{1}{\ell(\ell+1)}, \frac{1}{(\ell+2)(\ell-1)}, \frac{(\ell-2)!}{(\ell+2)!}\right\}$ respectively for $k=\{0,1,2,3\}$.

5. Determination of the completion amplitudes:

5.1 Circular equatorial orbits in Kerr (cont'd)

Step 6: Evaluate sums over ℓ via term-by-term differentiation of the closure relation

$$\sum_{\ell=2}^{\infty} {}_2Y_\ell(\theta)\,{}_2Y_\ell(\theta_0) = (2\pi)^{-1}\delta(\cos\theta - \cos\theta_0),$$

Step 7: Obtain (dropping distributions supported on $\theta = \theta_0 = \pi/2$)

$$[\mathcal{I}_1^{\rm rec}]\,(\theta; r_0) = -\frac{2(r_0^2 + a^2\cos^2\theta)\left[(r_0^2+5a^2)\,E\ - 3a\,L\,\right]}{3M\Delta_0^2},$$

$$[\mathcal{I}_2^{\rm rec}]\,(\theta; r_0) = +\frac{2(r_0^2 + a^2\cos^2\theta)\,[6\,L\ - a\,E\,(9-\cos 2\theta)]}{6aM\sin^2\theta}.$$

5. Determination of the completion amplitudes:
5.1 Circular equatorial orbits in Kerr (cont'd)

Step 8: Write **completion** piece of the metric perturbation as

$$h_{\alpha\beta}^{\rm comp\pm} = \mathcal{E}^{\pm}\,\frac{\partial g_{\alpha\beta}^{\rm Kerr}(x^\mu; M, J)}{\partial M} + \mathcal{J}^{\pm}\,\frac{\partial g_{\alpha\beta}^{\rm Kerr}(x^\mu; M, J)}{\partial J}$$

(where $J = Ma$) with amplitudes $\mathcal{E}^{\pm}$, $\mathcal{J}^{\pm}$ to be determined.

Step 9: Construct the corresponding contributions to $\mathcal{I}_1$ and $\mathcal{I}_2$, and obtain their jumps:

$$\left[\mathcal{I}_1^{\rm comp}\right](\theta; r_0) = +\frac{2(r_0^2 + a^2\cos^2\theta)\left[(r_0^2 + 5a^2)[\mathcal{E}] - 3a[\mathcal{J}]\right]}{3M\Delta_0^2},$$

$$\left[\mathcal{I}_2^{\rm comp}\right](\theta; r_0) = -\frac{2(r_0^2 + a^2\cos^2\theta)\left[6[\mathcal{J}] - a[\mathcal{E}](9 - \cos 2\theta)\right]}{6aM\sin^2\theta},$$

where $[\mathcal{E}] := \mathcal{E}^+ - \mathcal{E}^-$ and $[\mathcal{J}] := \mathcal{J}^+ - \mathcal{J}^-$.

5. Determination of the completion amplitudes:
5.1 Circular equatorial orbits in Kerr (cont'd)

Step 10: The requirement

$$[\mathcal{I}_n]\,(\theta) = [\mathcal{I}_n^{\rm rec}]\,(\theta) + [\mathcal{I}_n^{\rm comp}]\,(\theta) \equiv 0,$$

for $n = 1$ and $n = 2$, now gives

$$[\mathcal{E}] = E, \qquad [\mathcal{J}] = L.$$

(If fact, this follows from $[\mathcal{I}_2]\,(\theta) \equiv 0$ alone.)

5. Determination of the completion amplitudes:
5.1 Eccentric equatorial orbits in Kerr

Step 1: Express energy-momentum as superposition of "conserved" partial rings:

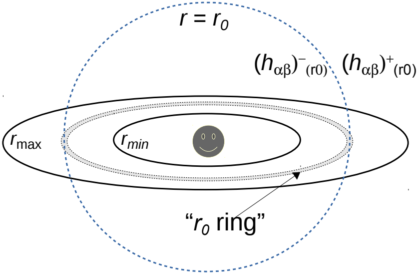
$$
\begin{aligned}
T_{\alpha\beta}^{\rm SAS} &= \frac{\mu u_\alpha(r) u_\beta(r)}{T_r r^2 \dot{r}(r)} \Theta(r - r_{\min})\Theta(r_{\max} - r)\delta(\cos\theta) \\
&= \frac{\mu}{T_r}\int_{r_{\min}}^{r_{\max}} dr_0 \left[\left(\frac{u_\alpha(r_0)u_\beta(r_0)}{r_0^2|\dot{r}(r_0)|} + \{\dot{r}\to -\dot{r}\}\right)\delta(r-r_0) + \underline{A_{\alpha\beta}(r)\delta'(r-r_0)}\right]\delta(\cos\theta)
\end{aligned}
$$

Step 2:
Correspondingly write source of Teukolsky eq as integral of r_0-ring contributions, and

$$\psi_4^{\rm SAS,\ell} = \int \psi^{\ell}_{4(r_0)} dr_0.$$

Introduce corresponding

$\Psi^{\pm}_{(r_0)}$, $(h^{\pm}_{\alpha\beta})_{(r_0)}$, $(\mathcal{I}^{\pm})_{(r_0)}$, $\mathcal{E}^{\pm}_{(r_0)}$, $\mathcal{J}^{\pm}_{(r_0)}$, etc.



5. Determination of the completion amplitudes:

5.1 Eccentric equatorial orbits in Kerr

Step 3:
Proceed for each r_0-ring as for a circular orbit to calculate $[\mathcal{I}_n^{\rm rec}]_{(r_0)}$ and $[\mathcal{I}_n^{\rm comp}]_{(r_0)}$.

Step 4: Impose continuity of the invariants for each r_0-ring,

$$[\mathcal{I}_n]_{(r_0)}(\theta) = [\mathcal{I}_n^{\rm rec}]_{(r_0)}(\theta) + [\mathcal{I}_n^{\rm comp}]_{(r_0)}(\theta) \equiv 0,$$

and solve for partial amplitudes $[\mathcal{E}]_{(r_0)}$ and $[\mathcal{L}]_{(r_0)}$.

Step 5: Get full jumps $[\mathcal{E}]$ and $[\mathcal{L}]$ by integrating over all partial rings:

$$[\mathcal{E}] = \int_{r_{\rm min}}^{r_{\rm max}} [\mathcal{E}]_{(r_0)} dr_0, \qquad [\mathcal{J}] = \int_{r_{\rm min}}^{r_{\rm max}} [\mathcal{J}]_{(r_0)} dr_0$$

Summary & Discussion

- We have confirmed earlier results for circular orbits, and are extending them to eccentric (equatorial) orbits.
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- Result is extremely simple. Can it follow from a simpler (but equally rigorous) argument?
- For orbits that start at infinity, result follows from simple considerations at infinity (noting completion amplitudes are time-independent).
- To do (1): Extension to generic orbits
- To do (2): "gauge smoothing" of residual discontinuity in metric across $\bar{\mathcal{S}}$.