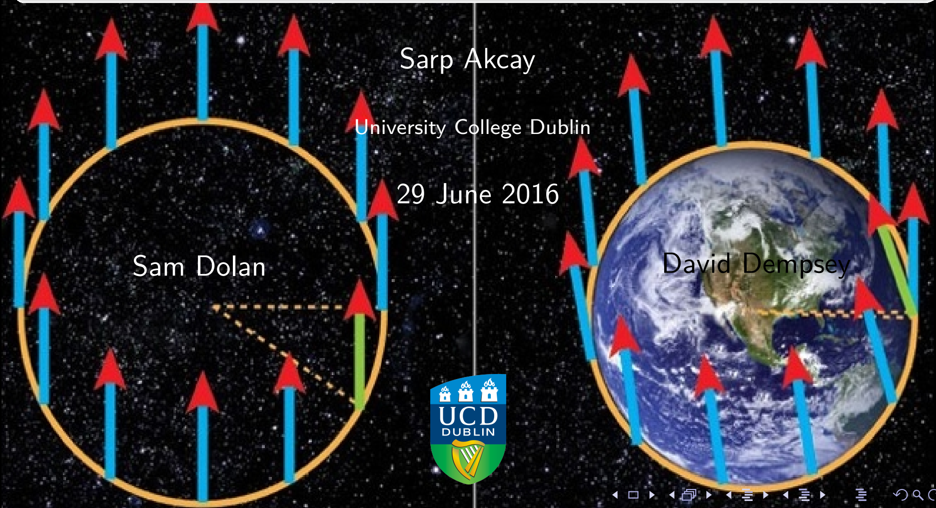
Self-force corrections to spin precession for eccentric orbits in Schwarzschild spacetime (again)

Sarp Akcay

University College Dublin

29 June 2016



Last time on Capra

Numerical Results for $\Delta\psi(e = 0.1)$

p	$\Delta\psi$	$\lim_{e\to 0} \Delta\psi$	$(\Delta\psi - \lim_{e\to 0} \Delta\psi)/e^2$
10	$-5.0374746183 \times 10^{-2}$	-5.06715×10^{-2}	2.96711×10^{-2}
20	$-4.105502714 \times 10^{-2}$	-4.12414×10^{-2}	1.8634×10^{-2}
30	$-2.97664644 \times 10^{-2}$	-2.99044×10^{-2}	1.3792×10^{-2}
40	$-2.309158147 \times 10^{-2}$	-2.32001×10^{-2}	1.085×10^{-2}
50	$-1.880823903 \times 10^{-2}$	-1.88976×10^{-2}	8.93×10^{-3}
60	$-1.58482057 \times 10^{-2}$	-1.59242×10^{-2}	7.595×10^{-3}
70	$-1.3686455636 \times 10^{-2}$	-1.37523×10^{-2}	6.588×10^{-3}
80	$-1.204035626 \times 10^{-2}$	-1.20966×10^{-2}	5.826×10^{-3}
90	$-1.074624358 \times 10^{-2}$	-1.07983×10^{-2}	5.204×10^{-3}
100	$-9.702092699 \times 10^{-3}$	-9.74941×10^{-3}	4.732×10^{-3}
110	$-8.84229541 \times 10^{-3}$	-8.88571×10^{-3}	4.341×10^{-3}
120	$-8.122988585 \times 10^{-3}$	-8.16224×10^{-3}	3.925×10^{-3}
130	$-7.511573415 \times 10^{-3}$	-7.54748×10^{-3}	3.59×10^{-3}
140	$-6.985103217 \times 10^{-3}$	-7.01869×10^{-3}	3.359×10^{-3}
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WRONG!

Last time on Capra

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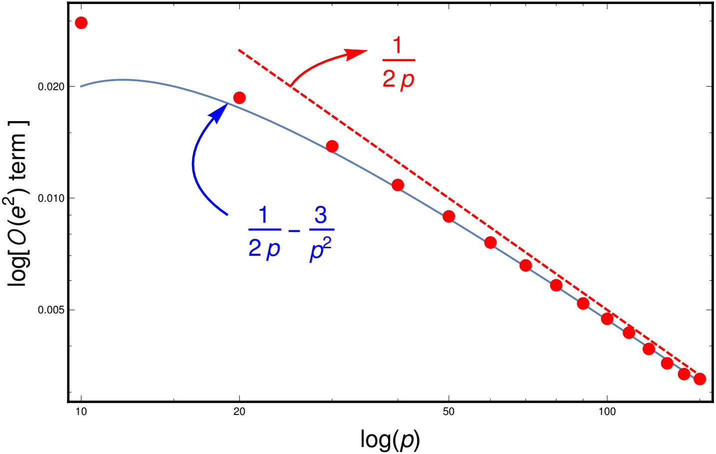
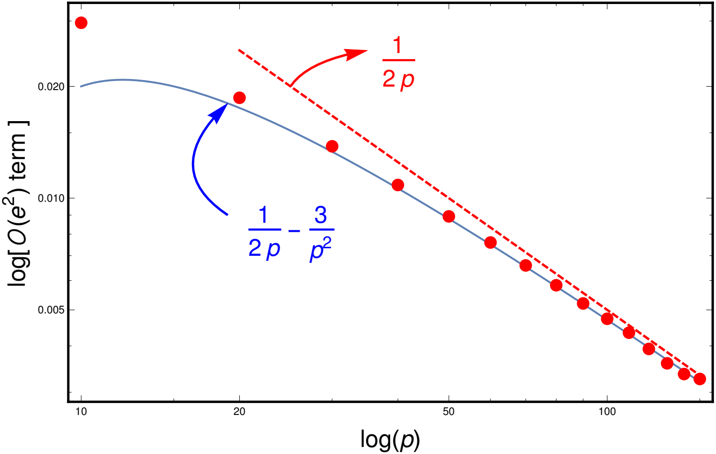


Last time on Capra

and $\Delta\psi - \lim_{e\to 0}\Delta\psi = \frac{1}{2}\frac{e^2}{p}$ vs. NO $1/p$ term (PN L.O.)



Recalling the formalism I

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ψ is the accumulated precession per angle between two frames.

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1. //-transported tetrad: $\dot{\lambda}_i^\alpha = 0 \quad \{u^\alpha, \lambda_1^\alpha, \lambda_2^\alpha \propto \hat{\theta}, \lambda_3^\alpha\}$
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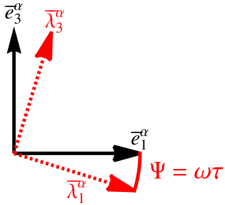
$$\begin{pmatrix} \lambda_1^\alpha \\ \lambda_3^\alpha \end{pmatrix} = \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix} \begin{pmatrix} e_1^\alpha \\ e_3^\alpha \end{pmatrix}$$



Using $\dot{\lambda}_1 = \dot{\lambda}_3 = 0$ we get $\dot{\Psi} = \boxed{e_{3\alpha}\,\dot{e}_1^\alpha = -\,e_{1\alpha}\,\dot{e}_3^\alpha}$

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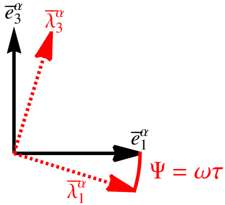

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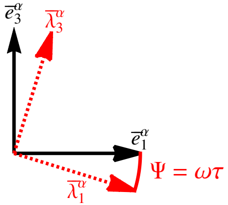

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Thus

$$\delta\dot{\Psi} = \delta\left(g_{\alpha\beta}\, e_3^\beta\, u^\gamma \nabla_\gamma e_1^\alpha\right) = -\delta\left(g_{\alpha\beta}\, e_1^\beta\, u^\gamma \nabla_\gamma e_3^\alpha\right)$$

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where

$$\left.\begin{aligned} \delta\Psi &= \int_0^{2\pi} \frac{d\bar{\Psi}}{d\chi}\left(\frac{\delta\dot{\Psi}}{\dot{\bar{\Psi}}} - \frac{\delta\dot{R}}{\bar{u}^r}\right) d\chi \\ \delta\Phi &= \int_0^{2\pi} \frac{d\bar{\Phi}}{d\chi}\left(\frac{\delta L}{L_0} - \frac{\delta\dot{R}}{\bar{u}^r}\right) d\chi \end{aligned}\right\}$$

$\delta\Phi$: Barack & Sago 2011

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Progress since Capra 2015: missing terms

The crux: numerical computation of

$$\delta\dot{\Psi} = \underbrace{\delta\left(g_{\alpha\beta}\, e_3^\beta\, u^\gamma \nabla_\gamma\, e_1^\alpha\right)}_{\delta\dot{\Psi}_1} = \underbrace{-\delta\left(g_{\alpha\beta}\, e_1^\beta\, u^\gamma \nabla_\gamma\, e_3^\alpha\right)}_{\delta\dot{\Psi}_2},$$

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$$\delta\dot{\Psi}_1 = \frac{1}{2}\dot{\bar{\Psi}}\left(h_{uu} + h_{33} - h_{11}\right) + \; c_{01}\bar{\Gamma}_{311} \; + \; c_{03}\bar{\Gamma}_{313} \; + \delta\Gamma_{310} \quad + \frac{dc_{13}}{d\tau} \;\; ,$$

$$\delta\dot{\Psi}_2 = \frac{1}{2}\dot{\bar{\Psi}}\left(h_{uu} + h_{11} - h_{33}\right) - \; c_{01}\bar{\Gamma}_{131} \; - \; c_{03}\bar{\Gamma}_{133} \; - \delta\Gamma_{130} \quad - \frac{dc_{31}}{d\tau} \;\; ,$$

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Missing terms: $\{c_{01}, c_{03}\} \times \quad \text{func.}(p, e, \chi)$

E.g. $\quad c_{03}(\chi) = \text{func.}(p, e, \chi) \times \int_0^\chi \frac{d\tau}{d\chi'}\, F_\phi^{\text{cons}}(\chi')\, d\chi'$

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$\Rightarrow$ Integrand is zero or integrates to zero.

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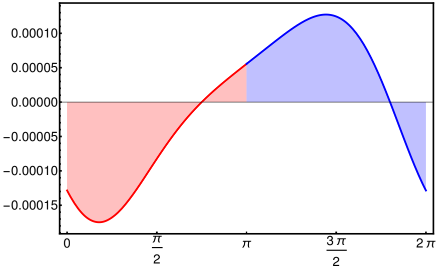

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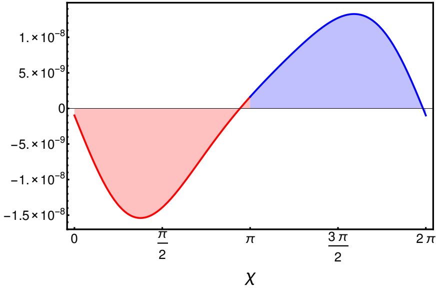

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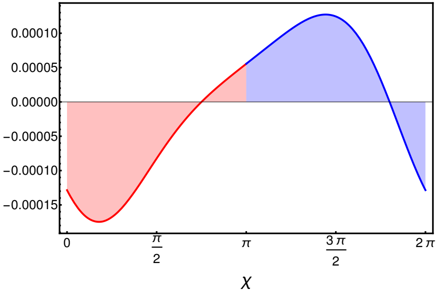

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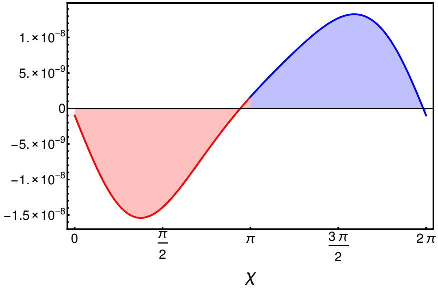

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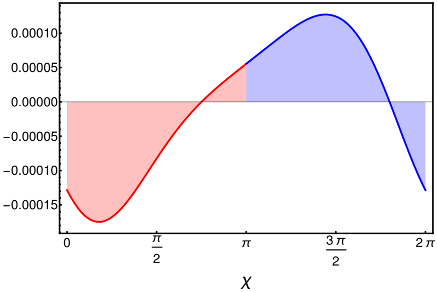

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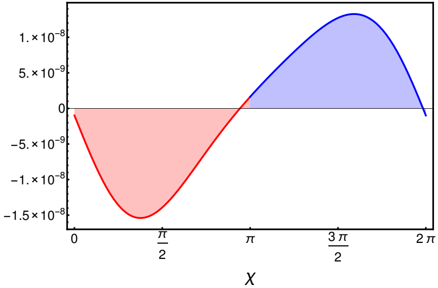

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$\delta\Psi_1 - \delta\Psi_2 \lesssim \mathcal{O}(10^{-10})$ for $p \in [10, 100]$, $e \leq 0.2$

Numerical results

Data for $p = \{10, 20, 30, \ldots, 100\}$, $e = \{0.05, 0.1, 0.15, 0.2\}$

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Difference is gauge invariant and
its physical origin understood (fixing two frequencies).

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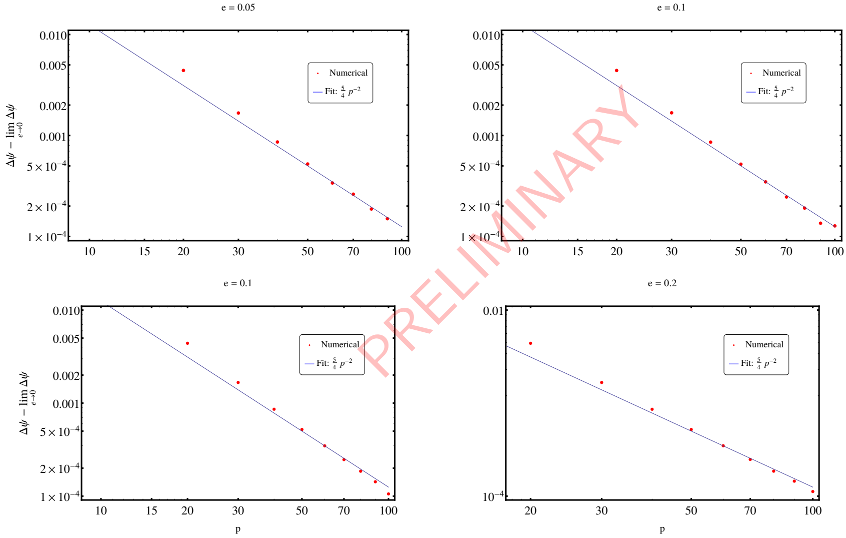
Extract the $\mathcal{O}(e^2)$ term $\sim \frac{5}{4}\frac{e^2}{p^2}$. Compare with future 1PN result.



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Consistent with current PN knowledge
i.e. NO p^{-1} terms at $\mathcal{O}(e^2)$.

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- Eccentric, tidal gauge invariants.